

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER

CLASS XII

TIME: 3hrs

MATHEMATICS

M.M 100

GENERAL INSTRUCTIONS:

1. All questions are compulsory
2. The question paper consists of 29 questions divided into three sections A, B and C.
Section A comprises of 10 questions of 1 mark each. Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
3. All questions in Section A are to be written in one word, one sentence or as per the exact requirement of questions.
4. There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION- A

1. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the determinant $A^2 - 2A$.
2. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, find $(BA)^T$
3. If $f: R \rightarrow R$ is a bijection given by $f(x) = x^3 + 3$, find $f^{-1}(x)$.
4. Using principal value, evaluate: $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$
5. Find the equation of the tangent to the curve $y = -5x^2 + 6x + 7$ at the point $\left(\frac{1}{2}, \frac{35}{4}\right)$
6. Find $|\vec{a} - \vec{b}|$, if two vectors \vec{a} and \vec{b} are such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$.
7. Evaluate: $\int_0^{\pi} \frac{dx}{1 + \sin x}$
8. Evaluate: $\int \frac{3x}{1 + 2x^4} dx$
9. Let $\begin{bmatrix} x + 3 & 2x \\ 6 & y \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 6 & 3 \end{bmatrix}$, then find the value of x and y.
10. Find projection of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ on $\vec{b} = \hat{i} - 3\hat{j} + 2\hat{k}$

SECTION B

11. Let $A=Q \times Q$. Let $*$ be a binary operation on A defined by : $(a,b)*(c,d) = (ac, ad+bc)$. Find i) Identity element of $(A,*)$ & ii) the invertible element of $(A,*)$.
12. Solve: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$, $x > 0$

OR

Find the value of $2\tan^{-1}(\frac{1}{5}) + \sec^{-1}(\frac{5\sqrt{2}}{7}) + 2\tan^{-1}(\frac{1}{8})$

13. Using properties, show that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

14. Show that $f(x) = \begin{cases} \frac{\sin 3x}{\tan 2x}, & \text{if } x > 0 \\ \frac{3}{2}, & \text{if } x = 0 \\ 3 \frac{(e^x - 1)}{e^{2x} - 1}, & \text{if } x < 0 \end{cases}$ is continuous at $x=0$

15. If $y = [x + \sqrt{x^2 + a^2}]^n$, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

OR

If $y = A \cos mx + B \sin mx$, show that $\frac{d^2y}{dx^2} + m^2y = 0$

16. Show that $y = \log(1+x) - \frac{2x}{2+x}$ is an increasing function of x for all values of $x > -1$

17. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{2\cos x + 4\sin x}$ OR Evaluate: $\int \frac{3x-2}{(x+3)(x^2+1)} dx$

18. Solve the differential equation : $\frac{dy}{dx} = 3y \cot x + \sin 2x$, given that $y=2$, when $x = \frac{\pi}{2}$.

OR

Solve the differential equation: $(1+y^2)(1+\log x)dx + xdy = 0$, given that $y = 1$, when $x = 1$

19. Solve the differential equation: $(1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} (1 - \frac{x}{y}) dy = 0$

20. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors

$2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda \hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ

21. Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$$

22. Three students A, B and C are given a question to solve. Their respective probability to solve the question are $\frac{1}{2}$, $\frac{1}{3}$ & $\frac{1}{4}$. What is the probability that only one of them solves it correctly?

SECTION C

23. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the following system of linear equations $2x-3y+5z=11$, $3x+2y-4z=5$ and $x+y-2z=-3$

24. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

25. Evaluate: $\int_0^a \text{Sin}^{-1} \left(\sqrt{\frac{x}{a+x}} \right) dx$ OR Evaluate: $\int_0^\pi \frac{x \text{Sin} x}{1+\text{Cos}^2 x} dx$

26. Find the area of the region: $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

27. Find the equation of the plane passing through the point $(-1, 2, 1)$ and perpendicular to the line joining the points $(-3, 1, 2)$ and $(2, 3, 4)$. Find also the perpendicular distance of the origin from the plane.

28. If an old man rides his motor cycle at 25 km/hr, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5 per km. He has Rs 100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.

29. There are 3 bags each containing 5 white balls and 3 black balls. Also there are 2 bags, each containing 2 white balls and 4 black balls. A white ball is drawn at random. Find the probability that this white ball is from a bag of the first group.
