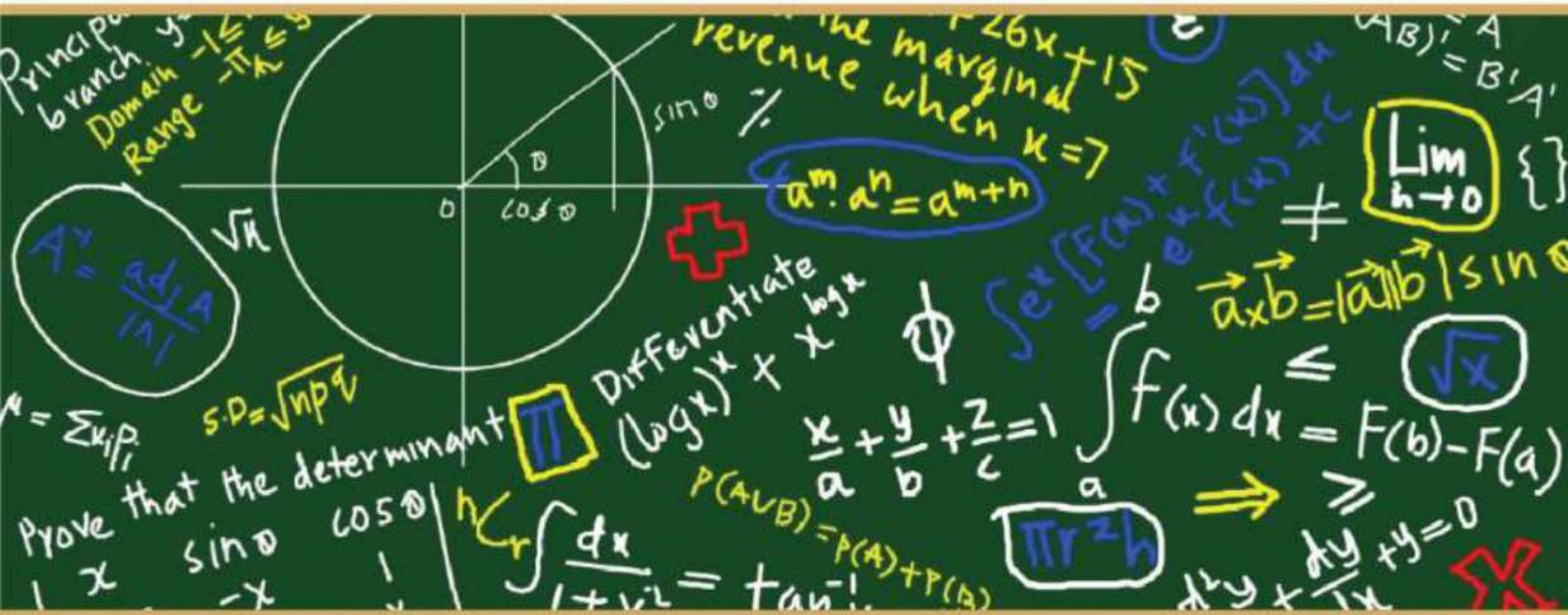
तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन



केंद्रीय विद्यालय संगठन क्षेत्रीय कार्यालय एर्णाकुलम

KENDRIYA VIDYALAYA SANGATHAN
REGIONAL OFFICE, ERNAKULAM

CLASS XII

STUDY MATERIAL

MATHEMATICS-[041]

Based on Latest CBSE Curriculum
Session 2022-23

OUR PATRON

HON. DEPUTY COMMISSIONER KVS RO ERNAKULAM REGION

SHRI R. SENTHILKUMAR

INSPIRATION

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Deputy Commissioner

केन्द्रीय विद्यालय संगठन

केन्द्रीय विद्यालय संगठन,
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F.31/Acad/KVS(EKM) **Date** : *14/09/2022*

MESSAGE FROM DEPUTY COMMISSIONER

It gives me immense pleasure to publish the study material for class XII Mathematics. I am sure that the support material will definitely be of great help to the class XII students of all Kendriya Vidyalayas of our region. .

This Students' Support Material has been prepared to improve their academic performance. This is a product of the combined efforts of a team of dedicated and experienced teachers with expertise in their subjects. This material is designed to supplement the NCERT text book.

The Support Material contains all the important aspects required by the students. Care has been taken to include the latest syllabus, summary of all the chapters, important formulae, Sample question papers, problem solving and case-based questions. It covers all essential components that are required for quick and effective revision of the subject

I would like to express my sincere gratitude to the in- charge Principal and all the teachers who have persistently striven for the preparation of this study material. Their selfless contribution in making this project successful is commendable.

"An ounce of practice is worth tons of knowledge", students will make use of this material meticulously to reap the best out of this effort.

With Best Wishes

(R Senthil Kumar)
Deputy Commissioner

PREFACE

A good education is one that teaches a student to think. Mathematics develops logic and skills of reasoning among students.Focus of this material is primarily to strengthen the mind to absorb the concepts and bring in the students the required self-confidence while learning the subject. Mathematics should be learnt with interest and it is made simple and approachable. This material is developed keeping in mind the latest CBSE curriculum. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in focus group on teaching of mathematics 2005 which is to meet the emerging needs of all categories of students. This material will definitely provide the students all essential components that are required for effective revision of the subject.

Hoping this material will serve the purpose of enhancing students' confidence level to help them perform better.

BEST OF LUCK !

Team of teachers who contributed

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CONTET INDEX

Revised Curriculum

CLASS-XII

(2023)

One Paper **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions **15 Periods**

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions **15 Periods**

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices **25 Periods**

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. On-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants **25 Periods**

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability 20 Periods

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, $like\ \sin^{-1} x\ , \cos^{-1} x \text{ and } \tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.
Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives 10 Periods

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals 20 Periods

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{\mathrm{dx}}{\mathrm{x}^2 \pm \mathrm{a}^{2,}} \int \frac{\mathrm{dx}}{\sqrt{\mathrm{x}^2 \pm \mathrm{a}^2}}, \int \frac{\mathrm{dx}}{\sqrt{\mathrm{a}^2 - \mathrm{x}^2}}, \int \frac{\mathrm{dx}}{\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}}, \int \frac{\mathrm{dx}}{\sqrt{\mathrm{ax}^{2+\mathrm{bx}+\mathrm{c}}}}$$

$$\int \frac{\mathrm{px} + \mathrm{q}}{\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}} \mathrm{dx}, \int \frac{\mathrm{px} + \mathrm{q}}{\sqrt{\mathrm{ax}^{2+}\mathrm{bx} + \mathrm{c}}} \mathrm{dx}, \int \sqrt{\mathrm{a}^2 \pm \mathrm{x}^2}\, \mathrm{dx}, \int \sqrt{\mathrm{x}^2 - \mathrm{a}^2}\, \mathrm{dx}$$

$$\int \sqrt{ax^2 + bx + c}\, dx,$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals 15 Periods

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations 15 Periods

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{where p and q are functions of x or constants.}$$

$$\frac{dx}{dy} + px = q, \text{where p and q are functions of y or constants.}$$

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors 15 Periods

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry 15 Periods

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming 20 Periods

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability 30 Periods

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes’ theorem, Random variable and its probability distribution, mean of random variable.

CHAPTER 1

RELATIONS AND FUNCTIONS

GIST OF THE LESSON

1. If A≠∅ and B≠∅ then A×B= {(a, b): a∈ A and b∈ B} is called Cartesian Product of sets A and B. the element (a, b) is called ordered pair
2. A=∅ or B=∅ then A×B=∅
3. n(A×B) = n(A)×n(B)
4. If A≠∅ and B≠∅ then a set R is said to be a relation from A to B if R⊂ A×B
5. Number of relations that can be defined from A to B is $2^{n(A)\times n(B)}$
6. Let A≠∅ then a set R is a relation on A if R⊂ A×A
7. Notation aRb⇔(a,b)∈R
8. ∅⊂A×A is a relation on A known as empty relation or void relation or null relation
9. A×A⊂A×A is a relation on A known as universal relation on A
10. If R= {(a, a): a ∈A} known as identity relation on A
11. R is a reflexive relation on A if (a, a) ∈R for every a ∈A
    or R is a reflexive relation on A if aRa for every a ∈A
12. R is a symmetric relation on A if (a, b) ∈R => (b, a) ∈R for every a, b∈ A
    Or R is a symmetric relation on A if aRb =>bRa for every a, b∈ A
13. R is transitive on A if (a,b)∈R and (b, c)∈R=>(a, c)∈R for every a, b, c∈ A
    or R is transitive on A if aRb and bRc =>aRc for every a, b, c ∈A
14. R is an equivalence relation on A if it is reflexive ,symmetric and transitive
15. If R is an equivalence relation on A and a∈ A then equivalence class of a, [a]= {b∈ A:(b, a) ∈R}
16. Sets $A_1, A_2, A_3 \ldots A_n$ is a partition of set A if $A_i \cap A_j = \emptyset$ if i≠j and $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n = A$
17. Equivalence relation defined on a set gives a partition of the set as equivalence classes and every partition of set gives an equivalence relation
18. If A≠∅ and B≠∅ ,a function f: A->B is a relation which associate each element of A to a Unique element of B, A is known as domain of f, B is known as co domain of f
19. If f(a)=b then b is known as image of a and a is known as pre-image of b
20. Set of all images of elements of A is known as range of f, Range of f ⊂B
21. A function f: A->B is one to one or injective if a≠ b =>f(a)≠f(b) ∀ a, b∈ A
    or f: A->B is one to one or injective if f(a)=f(b)=>a=b ∀ a, b∈ A
22. A function which is not one to one is known as many to one
23. A function f: A->B is onto or surjective if for each element b∈ B, there exists a∈ A such that f(a)=b
24. A function f: A->B is onto or surjective if Range of f=B
25. A function f: A->B which is not onto is known as into function
26. A function f: A->B which is both one to one and onto is known as a bijection
27. A function f: A->B which is both injective and surjective is known as a bijection

28. If n(A)=m and n(B)=n then

    (i)Number of functions that can be defined from A to B =n^m

    (ii)Number of one-to-one functions from A to B =$\frac{n!}{[n-m]!}$ If n≥m otherwise it is zero

    (iii)Number of onto functions from A to B=$\sum_{r=1}^{n}(-1)^{n-r}\ {}^{n}C_r\ r^m$ if n≤m otherwise it is zero

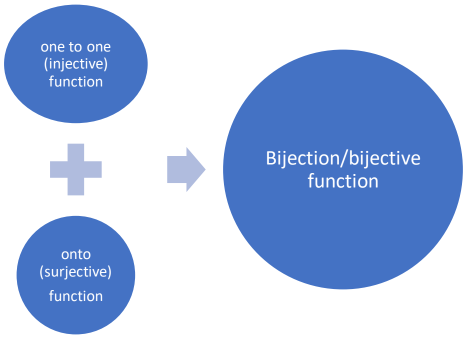
    (iv)Number of bijections from A to B = n! if n=m otherwise it is zero

29. For a finite set A, if a function f: A->A is one to one then f is onto
30. For a finite set A, if a function f: A->A is onto then f is one to one
31. For a finite set A the number of bijection from A to A=number of onto functions from A to A=number of one to one function from A to A=n!
32. Graphical test for a function: if any straight line parallel to y axis does not cut the graph at more than one point then the graph represents a function
33. Graphical test for one-to-one function: if any straight line parallel to x axis does not cut the graph at more than one point then the graph represents a one-to-one function
34. A function f: A->B is invertible if and only if it is a bijecti

CONCEPT MAPPING

Relations and functions		
Equivalence relation Equivalence relation on a nonempty set A is relation which is reflexive, symmetric and transitive		**Bijections** A function F: A->B is a bijection if it is both one to one and onto
R is reflexive on A if (a,a)∈R ∀a∈ A Or aRa ∀a∈ A		**One to one or injective functions** A function F: A->B is one to one or injective If a≠ b=> F(a)≠F(b) ∀a, b ∈A Or if F(a)=F(b) =>a=b ∀a, b ∈A
R is symmetric on A if (a, b) ∈R => (b, a) ∀a, b ∈A or aRb=>bRa ∀a, b ∈A		**Onto or surjective functions** A function F: A->B is a onto or surjective If for every b∈ B there exists a∈ A such that f(a)=b Or Range of f = B
R is transitive on A if (a, b) ∈R and (b, c) ∈ R=>aRc ∀a, b, c ∈ A or aRb and bRc =>aRc ∀a, b, c ∈A		**Many one function** A function F: A-> B is many one if it is not one to one
Equivalence class of a=[a]		**Into functions** A function F: A-> B is into function if it is not onto

If R is an equivalence relation on A and a∈ A then Equivalence class of a=[a]=set of all elements of A related a = {b ∈Aa, b) ∈R}		



Multiple choice questions		
1.	Let X= {1,2,3} and R is relation defined in the set X as R = {(1,3), (2,2), (3,2)} then the minimum ordered pairs should include in relation R to make it reflexive and symmetric is	
	a) (1,1), (2,3) and (1,2)	b) (3,3), (3,1), and (1,2)
	c) (1,1), (3,3), (3,1), and (2,3)	d) (1,1), (3,3), (3,1), and (1,2)
2	A relation R is defined as follows on N. Which of the following is reflexive relation	

	(a) R= {(x, y):x>y, x, y ϵN}	(b) R= {(x, y):x+ y=10, x, yϵ N}
	© R= {(x, y):x+4y=10, x, y ϵN}	(d) R= {(x, y): xy is a square number x, yϵ N}
3	The number of equivalence relations that can be defined in the set A= {1,2,3} which containing the elements (1,2) is	
	(a) 0	(b) 1
	(c) 2	(d) 3
4	A relation R is defined on Z as aRb if and only if a^2-7ab + 6b^2=0 then R is	
	a) Reflexive and symmetric	b) symmetric and not Reflexive
	c) transitive but not Reflexive	d) Reflexive but not symmetric
5	A relation R is defined on Z as aRb if and only if a-b+ √2 is an irrational number then R is	
	a) Reflexive	b) symmetric and Reflexive
	c) transitive and Reflexive	d) none of these
6	Let X= $\{x^2 : x \in N\}$ and the function f: N->X is defined as f(x)=x^2, x ∈ N then the function f is	
	a) Bijective	b) Not bijective
	c) Surjective only	d) Injective only
7	Which of the following function f: Z->Z is a bijection	
	a) f(x)=x^3	b) f(x)=x+2
	c) f(x)=2x+1	d) f(x)=$x^2 + 1$
8	Let f: R->R defined as f(x)=4+3cosx then f(x) is	
	a) Bijective	b) One to one but not onto
	c) Onto but not one to one	d) Neither one to one nor onto
9	The number of one-to-one functions that can be defined from the set {1,2,3,4,5} to {a, b}	
	a) 5	b) 0
	c) 2	d) 3
10	If F: N->N f(x)=$\begin{cases} \frac{n+1}{2} & if \quad n\ is\ odd \\ \frac{n}{2} & if \quad n\ is\ even \end{cases}$ then F(x) is	

	a) Bijective	b) One to one but not onto
	c) Onto but not one to one	d) Neither one to one nor onto

	Short Answer questions
11	Check whether the relation R on the set N of natural numbers given by R= {(a, b): b is a multiple of a } is reflexive, symmetric and transitive
12	Let W denote the set of words in English dictionary. Define the relation R by R= {(x, y):x, y ∈W such that x and y have at least one letter in common}. Show that this relation R is reflexive and symmetric but not transitive
13	An equivalence relation R in the set A divides it into equivalence classes A_1, A_2, A_3 Find (i) $A_1 \cup A_2 \cup A_3$ (ii) $A_1 \cap A_2 \cap A_3$
14	Check whether the relation R on set of all real numbers **R** as R= {(a, b): a $\leq b^3$} is reflexive, symmetric and transitive
15	Let R be a relation defined on the set of natural numbers N as R= {(x, y):x, y∈N,2x+y=11}. Verify whether R is reflexive, symmetric and transitive
16	If F= {(1,2), (2,4), (3,1), (4, k)} is a one -to-one function from set A to A, where A= {1,2,3,4} then find the value of k, also find the number of bijections can defined from A to A
17	A relation f defined in the set of real numbers R as f = {(a, b):$\sqrt{a}$ =b} Verify whether f is a function from R to R.
18	Show that the function f: R->R given by f(x)=$4x^3$+7 is a bijection
19	Let F: [2, ∞) ->B be a function defined as F(x)=5-4x+x^2 is a bijection then find B
20	Let f: R- {$\frac{-4}{3}$} -> R- {$\frac{4}{3}$} given by f(x) =$\frac{4x+3}{3x+4}$ Show that f is a bijective function
	LONG ANSWER QUESTIONS
21	Show that the relation on the set A={x∈Z:0≤x≤12} given by R= {(a, b):$\|a-b\|$is divisible by 4} is an equivalence relation Find all elements related to 1, equivalence class [1]
22	Prove that the relation R in the set Z of integers defined as R= {(a, b): a +b is divisible by 2} is an equivalence relation. Write the equivalence class[0]
23	Let N be the set of natural numbers and R be the relation on NXN defined by (a, b) R (c, d) if only if ad=bc for all a, b, c, d∈ N. Show that R is an equivalence relation

24	Let A= {1,2, 3, …,9} and R be the relation on A×A defined as (a, b) R (c, d) if and only if a+ d=b+ c. Prove that R is an equivalence relation also obtain the equivalence class [(2,5)]
25	Let R be the relation on N×N defined by (a, b) R (c, d) if and only if ad (b+ c) =bc (a+ d), Prove that R is an equivalence relation
26	Show that the relation R defined on the set N×N defined as (a, b) R (c, d) if and only if a^2+d^2=b^2+c^2 is an equivalence relation
27	Show that the function f: R ->R given by f(x)= $\frac{x}{x^2+1}$ is neither one to one nor onto
28	Show that f: N->N, given by f(x)=$\begin{cases} x+1 & if\ x\ is\ odd \\ x-1 & if\ xis\ even \end{cases}$ is a bijection
29	Show that the function f: N->N defined ad f(x)=$x^2 + x + 1$ is one to one but not onto
30	Let A = [–1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective (i) f (x) = $\frac{x}{2}$(ii) g(x) = $\|x\|$ (iii) h(x)=x$\|x\|$ (iv) k(x) = x^2.
CASE STUDY QUESTIONS	
31	 During a Swachh Bharat Abhiyan organizing committee wanted collect and segregate Metal, Paper, glass, batteries, organic and plastic waste. In the set of all participants a relation R defined as R= {(x, y) ∈R: both the participants x and y collect the same type of waste} Based on the information given above answer the following questions (a) Check whether R is an Equivalence relation in the set of all participants (b) In how many groups the participants are divided on the basis of their waste collection assume that there are participants to collect all type of waste (c) State whether the waste collected from different groups are segregated or not?

32	 A Physical education teacher asked the class teacher to form four teams with 12 members each out of the 48 students for a Kabaddi match.The class teacher asked to the students to form teams in such way that "two students are in the same team if difference of their roll numbers is divisible by 4" Based on the above information anser the following questions (a)Is it possible to form the teams by the method adopted by class teacher? (b) Which roll numbers are members of the team in which roll number 5 belongs? (c) If R is a relation defined in the set of roll numbers as R={(x,y):difference of x and y is divisible by 4 } show that R is an equivalence relation?
33	 Farmers plant sapling along straight lines parallel to each other as in figure .Let us assume that saplings are planted along the line y=x +1 and paralell to it.Let L be the set of all lines on the field Answer the following using the above information (i) R_1 be a relation defined on L as R_1={(l_1, l_2): l_1 ∥ l_2,where l_1, l_2∈L} then R_1 is …….. (a) Equivalence relation (b)only Reflexive (c)Not reflexive

	(c) Symmetric but not transitive (ii) Which of the following line is related the line y=x+1 as per definition of the relation R_1 (a) 2x-y+5=0 (b)2x+y=5 (c) 2x-2y=10 (d) x+y=1 (iii) R_2 be a relation defined on L as R_2={(l_1, l_2): l_1⍰ l_2,where l_1, l_2∈L} then R_2 is …….. (a) symmetric but neither reflexive nor transitive (b) reflexive and symmetric but not transitive © reflexive but neither symmetric not transitive (d) R is an equivalnce relation (iv) The function f:R->R defined by f(x)=x+1 is ……. (a) Injective but not surjective (b)Surjective but not injective (c) Bijective (d)Neither Injective Not Surjectie (v) Let function f:R->R defined by f(x)=x+1 then range of is (a) Q (b)Z ©W (d) R
34	

Sherlin and Danju are playing Ludo by rolling the dice alternatly,it was observed that the possible outcomes of the die belongs to the set B={1,2,3,4,5,6}.Let A ={S,D},be the set of all players

Answer the following questions

(i)Let R: B->B defined as R={(x,y): y is divisible by x} then R is

(a) reflexive and transitive but not symmetric

(b) reflexive and symmetric not transitive

© Not reflexive but symmetric and transitive

(d) Equivalence relation

(ii) How many relations can be defined from A to B

(a) 2^4

(b) 2^{36}

© 2^8

(d)2^{12}

(iii)How many functions can be defined from A to B

(a) 36

(b)64

© 720

(d)1024

(iv)Let R_1 be a relation on B defined as R_1={(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)} then R_1 is

(a) Symmetric

(b)Reflexive

(c)transitive

(d)None of these

(v) How many surjections can be defined from A to B

(a) 30

(b) 0

© 32

(d)64

35

In a Master chef competition final round 3 chef were selected and Judges assigned three dishes D= $\{D_1, D_2, D_{3,}\}$ to the participants P= $\{P_1, P_2, P_{3,}\}$ and asked them to prepare dishes as per the following rules.
Rule A: everybody has to prepare exactly one dish
Rule B: No two participant is allowed to prepare same dish
Rule C: All the dish must be prepared in the competition
Answer the following questions in the context of functions

(a) In how many ways all participants can choose a Dish as per rule A? Justify your answer
(b) In how many ways everybody can choose a dish to prepare as per Rule B? Justify your answer
©In How many ways all participants can prepare exactly one dish as per rule C, Justify your answer

ANSWERS

Q.No	Answer
1	c
2	d
3	c
4	d
5	a
6	a
7	b
8	d
9	b
10	c
11	Reflexive and transitive but not symmetric
12	Prove reflexive, symmetry and give example for not transitive
13	(i) $A_1 \cup A_2 \cup A_3 = A$ (ii) $A_1 \cap A_2 \cap A_3 = \emptyset$
14	Not reflexive, not symmetric and not transitive, give examples in each case
15	Not reflexive, not symmetric and not transitive, give examples in each case
16	K=3 and number of bijections =24
17	Not a function because no negative numbers have image in R

18	Prove 1to1 and onto
19	B=[1,∞)
20	Prove 1to1 and onto
21	Prove reflexive, symmetry and transitive [1]={1,5,9}
22	Prove reflexive, symmetry and transitive, [0]=set of all even integers
23	Prove reflexive, symmetry and transitive
24	Prove reflexive, symmetry and transitive, [(2,5)]={(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)}
25	Prove reflexive, symmetry and transitive
26	Prove reflexive, symmetry and transitive
27	Prove or give examples for not 1to 1 and not onto
28	Prove by taking different cases
29	Prove 1 to 1 and not onto
30	(i) one to one but not onto (ii) not one to one and not onto (iii) one to one and onto (iv) not one to one and not onto
31	It is an equivalence relation,6 groups, yes
32	(i)yes (ii){1,5,9,13,17,21,25,29,33,37,41,45} (iii) Prove reflexive, symmetry and transitive
33	(i)a (ii)c (iii)a (iv)c (v)d
34	(i)a (ii)d (iii)a (iv)d (v)b
35	(i)number of functions from D to P=27 (ii) Number of one to one function from D to P=3! =6 (iii) Number of onto functions from D to P=3! =6

CHAPTER 2
INVERSE TRIGONOMETRIC FUNCTIONS

BASIC CONCEPTS AND FORMULAE:
TRIGONOMETRIC FUNCTIONS

FUNCTIONS	DOMAIN	RANGE
Y=sin x	R	[-1,1]
Y = cos x	R	[-1,1]
Y = tan x	R$-(2n+1)\frac{\pi}{2}, n \in Z$	R
Y = cosec x	R$-n\pi, n \in Z$	R-(-1,1)
Y = sec x	R$-(2n+1)\frac{\pi}{2}$	R-(-1,1)
Y = cot x	R$-n\pi, n \in Z$	R

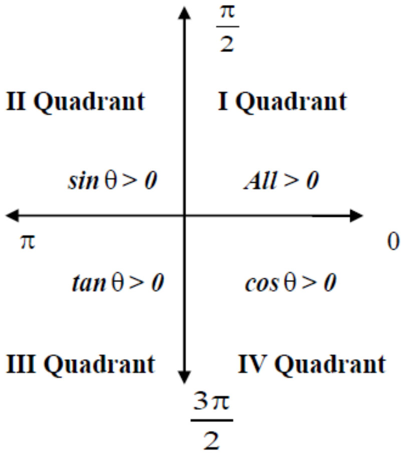
IMPORTANT TRIGONOMETRIC RESULTS & SUBSTITUTIONS

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$$
$$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$$
$$\sin(\pi \pm \theta) = \mp\sin\theta$$
$$\cos(\pi \pm \theta) = = \cos\theta$$
$$\tan(\pi \pm \theta) = \pm\tan\theta$$

$$\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta$$
$$\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$$
$$\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$$
$$\sin(2\pi \pm \theta) = \pm\sin\theta$$
$$\cos(2\pi \pm \theta) = \cos\theta$$
$$\tan(2\pi \pm \theta) = \pm\tan\theta$$



** **Sum and Difference formulae :**

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}, \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}, \quad \tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A},$$

$$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}, \quad \cot(A + B) = \frac{\cot A.\cot B - 1}{\cot B + \cot A} \quad \cot(A - B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}$$

$$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$
$$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

** Formulae for t-ratios of múltiple and sub-múltiple angles :

$$\sin 2A = 2 \sin A \cos A = \frac{2\tan A}{1+\tan^2 A}.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$1 + \cos 2A = 2\cos^2 A$ $\quad 1 - \cos 2A = 2\sin^2 A$ $\quad 1 + \cos A = 2\cos^2\frac{A}{2}$ $\quad 1 - \cos A = 2\sin^2\frac{A}{2}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, $\quad \tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$.

$\sin 3A = 3\sin A - 4\sin^3 A$, $\quad \cos 3A = 4\cos^3 A - 3\cos A$

$\sin 15° = \cos 75° = \frac{\sqrt{3}-1}{2\sqrt{2}}$. & $\cos 15° = \sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$,

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} = \cot 75°$ & $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3} = \cot 15°$.

$\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$ and $\cos 36° = \frac{\sqrt{5}+1}{4} = \sin 54°$.

$\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$ and $\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$.

$\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1 = \cot 67\frac{1}{2}^\circ$ and $\tan\left(67\frac{1}{2}\right)^\circ = \sqrt{2} + 1 = \cot\left(22\frac{1}{2}\right)^\circ$.

PRINCIPAL VALUE BRANCHES OF INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTIONS	DOMAIN	PRINCIPAL VALUE BRANCH
$Y=\sin^{-1}x$	[-1,1]	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$Y=\cos^{-1}x$	[-1,1]	$[0,\pi]$
$Y = \tan^{-1}x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$Y = \mathrm{cosec}^{-1}x$	R− (-1,1)	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$
$Y = \sec^{-1}x$	R − (-1,1)	$[0,\pi]-\{\frac{\pi}{2}\}$
$Y = \cot^{-1}x$	R	$(0,\pi)$

Values of trigonometric functions

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
Sin$x$	**0**	**1**	**0**	**-1**	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cos$x$	**1**	**0**	**-1**	**0**	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tan$x$	**0**	**n.d**	**0**	**n.d**	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\frac{\sqrt{3}}{1}$
Cosec$x$	**n.d**	**1**	**n.d**	**-1**	**n.d**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
Sec$x$	**1**	**n.d**	**-1**	**n.d**	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**

Cot$x$	**n.d**	**0**	**n.d**	**0**	**n.d**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$

Properties of Inverse trigonometric functions:

1. $\sin(\sin^{-1}x) = x, x \in [-1,1]$ *and* $\sin^{-1}(\sin x) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2. $(i)\ \sin^{-1}(\frac{1}{x}) = \text{cosec}^{-1}x \quad , x \geq 1\ or\ x \leq -1$
   (ii)$\cos^{-1}(\frac{1}{x}) = \sec^{-1}x\ , x \geq 1\ or\ x \leq -1$
   (iii)$\tan^{-1}(\frac{1}{x}) = \cot^{-1}x\ ,\ x > 0$
3. **(i)** $\sin^{-1}(-x) \quad =- \sin^{-1}x\ , x \in [-1,1]$
   (ii) $\tan^{-1}(-x) \quad =- \tan^{-1}x\ , x \in R$
   (iii)$\text{cosec}^{-1}(-x) \ =- \text{cosec}^{-1}x\ |x| \geq 1$

4. (i) $\cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1,1]$
   (ii)$\sec^{-1}(-x) \quad = \pi - \sec^{-1}x\ |x| \geq 1$
   (iii) $\cot^{-1}(-x) \quad = \pi - \cot^{-1}x\ , x \in R$

5. **(i)**$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ , x \in [-1\ , 1]$
   (ii)$\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}\ , x \in R$
   (iii)$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\ , |x| \geq 1$

MULTIPLE CHOICE QUESTIONS

1. Simplified form of $\cos^{-1}(4x^3 - 3x)$
   (a) $3\sin^{-1}x$ (b) $3\cos^{-1}x$ (c) $\pi - 3\sin^{-1}x$ (d) $\pi - 3\cos^{-1}x$
2. If $y = \sec^{-1}x$ then
   (a) $0 \leq y \leq \pi$ (b) $0 \leq y \leq \frac{\pi}{2}$ (c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$
   (d) none of these
3. The value of $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$ is equal to
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d)$\frac{7\pi}{6}$
4. Principal value of $\cos^{-1}(-\frac{\sqrt{3}}{2})$
   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{7\pi}{6}$
5. The principal value of $\sin^{-1}(\sin\frac{7\pi}{6})$
   (a) $\frac{\pi}{6}$ (b) $\frac{7\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) $\frac{\pi}{3}$
6. $Choose\ correct\ option$: $\sin(\tan^{-1}x)$ =……… when $|x| < 1$
   $(a)\ \frac{x}{\sqrt{1-x^2}}$ $(b)\ \frac{1}{\sqrt{1-x^2}}$ $(c)\ \frac{1}{\sqrt{1+x^2}}$ $(d)\frac{x}{\sqrt{1+x^2}}$

SHORT ANSWER TYPE QUESTIONS

7. Find the value of expression : $2\sec^{-1}2+\sin^{-1}(\frac{1}{2})$
8. Express in simplest form : $\tan^{-1}(\frac{\sin x}{1+\cos x})$, $-\pi < x < \pi$
9. Express in simplest form : $\tan^{-1}(\frac{\cos x}{1-\sin x})$ where $-\frac{3\pi}{2} < x < \frac{\pi}{2}$
10. Express in simplest form : $\tan^{-1}(\frac{\cos x}{1+sinx})$ where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
11. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ $,then\ \ find\ the\ value\ of$ $\cos^{-1}x+\cos^{-1}y$
12. Express in simplest form $\sin^{-1}(\frac{\sin x+\cos x}{\sqrt{2}})$, where $\frac{-\pi}{4} < x < \frac{\pi}{4}$
13. Find the value of $\sin[\frac{\pi}{3}-\sin^{-1}(-\frac{1}{2})]$
14. Write the principal value of $\tan^{-1}(1)+\cos^{-1}(-\frac{1}{2})$
15. Write the value of $\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$
16. Write $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$, $|x|>1\ in\ simplest\ form$
17. Express in simplest form : $\tan^{-1}(\frac{\cos x-\sin x}{\cos x+\sin x})$,$-\frac{\pi}{4} < x < \frac{\pi}{4}$
18. Write in simplest form $\tan^{-1}\{\frac{x}{\sqrt{a^2-x^2}}\}$,$-a < x < a$
19. Write in simplest form :$\tan^{-1}\sqrt{\frac{a-x}{a+x}}$,$-a < x < a$
20. Write in simplest form: $\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}$

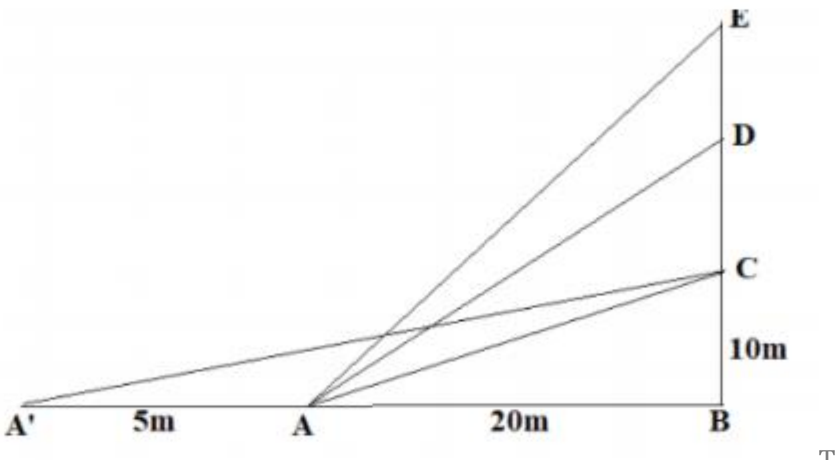
LONG ANSWER QUESTIONS

21. Write the simplest form of $\tan^{-1}(\frac{\sqrt{1-cosx}}{\sqrt{1+cosx}})$, where $-\pi < x < \pi$
22. Write in simplest form $\cot^{-1}(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}})$ where $x\epsilon(0,\frac{\pi}{4})$
23. Prove that $\tan^{-1}(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}})=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$, $\frac{-1}{\sqrt{2}} \le x \le 1$
24. Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$
25. Find the value of $\tan^{-1}[2\cos(2\sin^{-1}\frac{1}{2})]$
26. Prove that $\tan^{-1}[\frac{\sqrt{1+\cos x}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}]=\frac{\pi}{4}+\frac{x}{2}$, 0$< x < \frac{\pi}{2}$
27. Prove that $\tan^{-1}[\frac{\sqrt{1+cosx}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}]=\frac{\pi}{4}-\frac{x}{2}$ if $\pi < x < \frac{3\pi}{2}$
28. Prove that $\cot^{-1}[\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}]=\frac{\pi}{2}-\frac{x}{2}$, if $\frac{\pi}{2} < x < \pi$
29. Prove that $\tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}]=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2, -1 < x < 1$

CASE STUDY QUESTIONS

30. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID -19 protocol. Ram , Robert and Rahim are the three engineers who are working on this project . If “ A “ is considered to be a person viewing the hoarding board 20 metres away from the building , standing at the edge of a pathway nearby . Ram ,Robert and Rahim suggested to the film to place the

hoarding board at three different locations namely C,D and E . “C “ is at the height of 10 metres from the ground level. For the viewer A , the angle of elevation of “ D “is double the angle of elevation of “C” .The angle of elevation of “E” is triple the angle of elevation of “C” for the same viewer .Look at the figure given and based on the above information answer the following .



Type equation here.

1. Measure of $< CAB =$

   (a) $\tan^{-1}(2)$(b)$\tan^{-1}(\frac{1}{2})$ (c) $\tan^{-1}(1)$(d) $\tan^{-1}(3)$
2. Measure of <DAB =

   (a) $\tan^{-1}(\frac{3}{4})$(b)$\tan^{-1}(3)$ (c) $\tan^{-1}(\frac{4}{3})$(d)$\tan^{-1} 4$
3. Measure of $< EAB =$

   (a) $\tan^{-1}(11)$(b) $\tan^{-1}(3)$(c) $\tan^{-1}(\frac{2}{11})$(d) $\tan^{-1}(\frac{11}{2})$
4. If A’ is another viewer standing on the same line of observation across the road . If the width of the road is 5 metres , then the < CA’ B is

   (a)$\tan^{-1}(\frac{1}{2})$ b) $\tan^{-1}(\frac{1}{8})$ (c)$\tan^{-1}(\frac{2}{5})$ (d)$\tan^{-1}(\frac{11}{21})$
5. Also find the difference between $< CA\,B$ *and* <C A’B

   (a) $\tan^{-1}\left(\frac{1}{12}\right)$ (b) $\tan^{-1}\left(\frac{1}{8}\right)$ (c) $\tan^{-1}\left(\frac{2}{5}\right)$ (d) $\tan^{-1}(\frac{11}{21})$

31. .

A group of students of class XII visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Raj path (formerly called the Kingsway), is about 138 feet (42 metrs) in height.

1.What is the angle of elevation if they are standing at a distance of 42 m away from the monument ?

(a) $\tan^{-1} 1$

(b) $\sin^{-1} 1$

(c)$\cos^{-1} 1$

(d)$\sec^{-1} 1$

2.They want to see the tower at an angle of $\sec^{-1} 2$. So , they want to know the distance where they want to stand and hence find the distance .

(a)42 m (b) 20.12 m (c) 24.24 m (d) 24.64 m

3.If the altitude of the sun is at $\cos^{-1}(\frac{1}{2})$, then the height of the vertical tower that will cast a shadow of length 20 m is

(a) $20\sqrt{3}$ (b) $\frac{20}{\sqrt{3}}$ (c) $16\sqrt{3}$ (d)$\frac{16}{\sqrt{3}}$

4. The ratio of the length of a rod and its shadow is 1:2 . The angle of elevation of the sun is

(a) $\sin^{-1}\frac{1}{2}$(b) $\cos^{-1}\frac{1}{2}$(c)$\tan^{-1}\frac{1}{2}$ (d)$\cot^{-1}\frac{1}{2}$

5. Domain of $\sin^{-1} x$ is ……….

.(a)) (-1,1) (b) {-1, 1} (c) [-1, 1] (d) R

32.

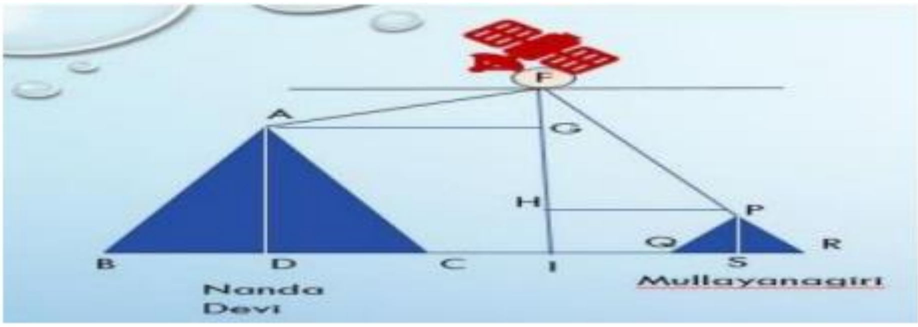
In the figure the angles of depression of the top and bottom of an 8 m tall building from the top of a multi storeyed building is $\tan^{-1}\frac{1}{\sqrt{3}}\,and\ \sec^{-1}\sqrt{2}$ respectively

1. The height of the multi-storeyed building is
   (a) $4(1+\sqrt{3)}\ \ m$ (b) 3(3+$\sqrt{3}$) m (c) 4(4+$\sqrt{3}$)m (d) 4(3+3$\sqrt{3}$)m
2. The distance between two building
   (a) 4(13+ $\sqrt{3)}$ m (b)4(31+ $\sqrt{3}$) m (c) 2(3+ $\sqrt{3}$) m (d) 4(3+ $\sqrt{3}$)m

3. The value of $\tan^{-1}(1/\sqrt{3)}$) is
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ $(b)\cos^{-1}\left(\frac{1}{2}\right)(c)\cos^{-1}(1/\sqrt{2)}$ (d) $\sin^{-1}(\frac{\sqrt{3}}{2})$
4. The value of $\sec^{-1}\sqrt{2}$ is
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ $(b)\cos^{-1}\left(\frac{1}{2}\right)$ $(c)\cos^{-1}1$ (d) $\tan^{-1}1$
5. The range of $\cos^{-1}x$
   (a) (0,π) (b)[0,π] (c) {0,π}(d) (0, π]

33.

A satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka , as
Nanda Devi (height 7816 metres) and Mullayanagiri (height1937 metres). The angles of depression from the satellite , to the top of Nanda Devi and Mullayanagiri are $\cot^{-1}\sqrt{3}$ and $\tan^{-1}\sqrt{3}$ respectively . If the distance between the peaks of the two mountains is 1937 km and the satellite is vertically above the midpoint of the distance between the two mountains . Look at the Figure given below and answer the Questions .



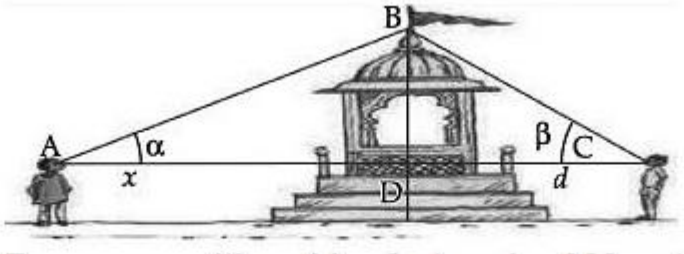
1.The distance of the satellite from the top of Nanda Devi hill is
(a)1139.4 km (b) 577.52km (c) 1937 km (d) 1025. 36 km

2. The distance of the satellite from the top of Mullayangiri is
(a) 1139.4 km (b) 577.52 km (c) 1937 km (d) 1025.36 km

3. The distance of the satellite from the ground is
(a) 1139.4 km (b) 577.52 km (c) 1937 km (d) 1025. 36 km

4 . What is the angle of elevation of the top of Nanda Devi if a man is standing at a distance of 7816 metre from Nanda Devi
(a) $\sec^{-1}(2)$ (b)$\cot^{-1}(1)(c)\sin^{-1}(\sqrt{3}/2\,)$ (d) $\cos^{-1}(\frac{1}{2})$

34.



Two men on either side of a temple of 30 m high observe its top at an angle of elevation α $and\ \beta$ respectively . (as in figure) . The distance between the two men is $40\sqrt{3}$ metres and distance between the first person A and the temple is 30 $\sqrt{3}$ metre .

1. Find α

   (a)$\sin^{-1}(\frac{\sqrt{3}}{2}$)(b) $\sin^{-1}(\frac{1}{2})$ (c) $\sin^{-1}(1/\sqrt{2}$) (d)$\sin^{-1}(1)$

2. Find β

   (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b)$\sin^{-1}(\frac{1}{2})$ (c) $\sin^{-1}(1/\sqrt{2}$) (d)$\sin^{-1}(1)$

3. If A is moving towards the temple and the distance between A and temple is 10 $\sqrt{3}$ m find the angle of elevation which A makes

   (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b)$\sin^{-1}(\frac{1}{2})$ (c)$\sin^{-1}\frac{1}{\sqrt{2}}$ (d) $\sin^{-1}(1)$

4 . Then what is the change in angle α

(a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$ (c) $\sin^{-1}(1/\sqrt{2}$)$\sin^{-1}(1)$

ANSWERS

1. (b) $3\cos^{-1}x$
2. (d) none of these
3. (c) $\frac{2\pi}{3}$
4. (c) $\frac{5\pi}{6}$
5. (c)$-\frac{\pi}{6}$
6. (d)$\frac{x}{\sqrt{1+x^2}}$
7. $\frac{5\pi}{6}$
8. $\frac{x}{2}$
9. $\frac{\pi}{4}+\frac{x}{2}$, $-\frac{\pi}{2}<\frac{\pi}{4}+\frac{x}{2}<\frac{\pi}{2}$
10. $\frac{\pi}{4}-\frac{x}{2}$, $where\ -\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$, so $0\leq\frac{\pi}{4}-\frac{x}{2}\leq\frac{\pi}{2}$
11. $\frac{\pi}{3}$
12. $x+\frac{\pi}{4}$
13. 1
14. $\frac{11\,\pi}{12}$
15. $\frac{\pi}{3}$

16. $\sec^{-1} x$
17. $\frac{\pi}{4} - x$
18. $\sin^{-1}(\frac{x}{a})$
19. $\frac{1}{2}\cos^{-1}(\frac{x}{a})$
20. $\tan^{-1}(\frac{x}{a})$
21. $\frac{x}{2}$
22. $\frac{x}{2}$

30 Case study Questions answers

1. (b) $\tan^{-1}(\frac{1}{2})$

2. (c) $\tan^{-1}(\frac{4}{3})$

3. $\tan^{-1}(\frac{11}{2})$

4. $\tan^{-1}(\frac{2}{5})$.

5. $\tan^{-1}(\frac{1}{12})$

31 Case study

1. (a) $\tan^{-1}(1)$, 2. (c) 24.24 m , 3. (a) $20\sqrt{3}$, 4. (c) $\tan^{-1}(\frac{1}{2})$ 5(c)[-1,1]

32 Case study

1. (a) 4(1+$\sqrt{3}$) m , 2. (d) 4(3+$\sqrt{3}$)m , 3. (a) $\sin^{-1}(\frac{1}{2})$, 4. (d) $\tan^{-1}(1)$,5. (b) $[0, \pi]$

33 Case study

1.(a)1139.4 km , 2. (c) 1937 km 3 . (b) 577. 52 km 4 . (b)$\cot^{-1}(1)$

34 Case study

1. (b)$\sin^{-1}(\frac{1}{2})$ 2. (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ 3. (a)$\sin^{-1}(\frac{\sqrt{3}}{2})$ 4. (b)$\sin^{-1}(\frac{1}{2})$

CHAPTER 3

MATRICES

CONCEPT MAPPING

- Definition of a matrix
- Order of a Matrix
- Construction of a Matrix of given order
- TYPES OF MATRICES

i) column matrix ii) row matrix iii) square matrix iv) diagonal matrix v) scalar matrix vi) identity matrix vii) zero or null matrix

- MATRIX OPERATIONS

  i) Addition

  ii)Subtraction

  iii)Scalar multiplication

  iv)Multiplication of matrices

- TRANSPOSE OF A MATRIX
- SYMMETRIC &SKEW –SYMMETRIC MATRICES
- INVERTIBLE MATRIX

GIST OF THE LESSON

MATRIX DEFINITION-

A matrix is an ordered rectangular array of numbers or functions

ROW MATRIX-

A matrix having only one row is called a row matrix

COLUMN MATRIX-

A matrix having only one column is called column matrix

SQUARE MATRIX

A matrix in which the number of rows is equal to the number of columns is called a square matrix. An m × n matrix is a square matrix if m=n

DIAGONAL MATRIX

A square matrix B = $[bij]_{mxm}$ is a diagonal matrix if all its non- diagonal elements are zero, that is if bij = 0, when i ≠ j.

SCALAR MATRIX

A diagonal matrix B = $[bij]_{mxm}$ is a scalar matrix if bij = 0, when i ≠ j and bij = k, when i = j, for some constant k

IDENTITY MATRIX

A square matrix A = [aij] n × n is an identity matrix if aij= 1,when $i = j$ aij =0,when $i \neq j$

ZERO MATRIX

A matrix is said to be zero matrix or null matrix if all its elements are zer

EQUALITY OF TWO MATRICES

Two matrices A = [aij] and B = [bij] are said to be equal if (i) they are of the same order (ii) each element of A is equal to the corresponding element of B, that is aij = bij for all i and j

ADDITION OF MATRICES

The sum of two matrices is a matrix obtained by adding the corresponding elements of the given matrices. If A = [aij] and B = [bij] are two matrices of the same order, say m × n. Then, the sum of the two matrices A and B is defined as a matrix C = [cij] m × n , where cij = aij + bij, for all possible values of i and j.

DIFFERENCE OF TWO MATRICES-

If A = [aij], B = [bij] are two matrices of the same order, say m × n, then difference A – B is defined as a matrix D = [dij], where dij = aij – bij, for all values of i and j

MULTIPLICATION OF A MATRIX BY A SCALAR

A= [aij] m × n is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

PROPERTIES OF MATRIX ADDITION

1) Commutative Law - If A = [aij], B = [bij] are matrices of the same order, say m × n, then A + B = B + A.
2)Associative law- For any three matrices A = [aij], B = [bij], C = [cij] of the same order, say m × n, (A + B) + C = A + (B + C)
3)Existence of additive identity- Let A = [aij] be an m × n matrix and O be an m × n zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.
4)The existence of additive inverse -Let A = [aij] m × n be any matrix, then we have another matrix as – A =- [aij] m × n such that A + (– A) = (– A) + A= O. Then -Ais the additive inverse of A or negative of A.

PROPERTIES OF SCALAR MULTIPLICATION OF A MATRX

If A = [aij] and B = [bij] be two matrices of the same order, say m × n, and k and l are scalars ,then (i)k(A +B) = k A + kB, (ii) (k + l)A = k A + l A

MULTIPLICATION OF TWO MATRICES

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let A = [aij] be an m × n matrix and B = [bjk] be an n × p matrix. Then the product of the matrices A and B is the matrix C of order m × p. To get the (i, k)th element c_{ik} of the matrix C, we take the i th row of A and kth column of B, multiply them elementwise and take the sum of all these products.

Non-commutativity of multiplication of matrices

Even if AB and BA are both defined, it is not necessary that AB = BA

PROPERTIES OF MULTIPLICATION OF MATRICES

1) The associative law : For any three matrices A, B and C. We have (AB) C = A (BC), whenever both sides of the equality are defined.
2) The existence of multiplicative identity: For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.
3) The distributive law : For three matrices A, B and C. A (B+C) = AB + AC (ii) (A+B) C = AC + BC

TRANSPOSE OF A MATRIX

If A = [aij] be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A′ or A^T.

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For any matrices A and B of suitable orders,

(i) (A′)′ = A

(ii) (A + B)′ = A′ + B′

iii) (A - B)′ = A′ - B′

(iv) (A B)′ = B′ A′

SYMMETRIC AND SKEW SYMMETRIC MATRICES

A square matrix A = [aij] is said to be symmetric if A′ = A, that is, [aij] = [aji] for all possible values of i and j.

A square matrix A = [aij] is said to be skew symmetric matrix if A′ = – A, that is aji = – aij for all possible values of i and j. Now, if we put i = j, we have aij = – aji. Therefore 2aii = 0 or aii = 0 for all i's. This means that all the diagonal elements of a skew symmetric matrix are zero.

For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix

Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrices

INVERTIBLE MATRICES

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible

MULTIPLE CHOICE QUESTIONS

Question NO	Question	
1	If order of matrix A is 2x3 and order of matrix B is 3x4 , find the order of AB. 1) 2x4 2) 2x2 3) 4x2 4) 3x3	
2	If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then A+A′ is 1) skew –symmetric 2) symmetric 3) diagonal matrix 4) zero matrix	
3	The number of all possible matrices of order2x3 with entry 1 or 2 1)16 2) 64 3) 6 4) 24	

4 If a matrix A is both symmetric and skew –symmetric , then A is necessarily a

1) diagonal matrix 2) zero square matrix
3) square matrix 4) Identity matrix

5 If A = [aij] is a square matrix of order 2 such that

aij = $\begin{cases} 1\ when\ i \neq j \\ 0 when\ i = j \end{cases}$, then A^2 is:

1) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$, 2) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, 3) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$,
4) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

SHORT ANSWER TYPE QUESTIONS

6 Show that $A A^{l}$ and $A^{l} A$ are both symmetric matrices for any matrix A.

7 Give an example of two non-zero 2×2 matrices A and B such that AB=0

8 If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, find a + b + c

9 If 2 A + 3 B = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3 A + 2 B = $\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$, find A and B.

10 Find the matrix A such that A $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ = $\begin{bmatrix} 0 & -3 \\ 10 & 3 \end{bmatrix}$.

11 Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as a sum of symmetric and skew symmetric matrices.

12 If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, f(x) = x^2 -2x – 3 , show that f(A) = 0.

13 If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the matrix K so that $A^2 = 8A + K$

14 If A and B are matrices of same order, then prove that $AB^{I} - BA^{I}$ is skew symmetric matrix.

15 If A is a square matrix such that $A^2 = I$. Then prove that $(A - I)^3 + (A + I)^3 - 7A = A$

16 **<u>CASE STUDY1</u>**

The following matrix depicts a number of students of a school who were awarded for discipline, attendance and obedience.

$$\begin{array}{cccc} \text{discipline} & \text{attendance} & \text{obedience} & \\ \left(\begin{matrix} 18 \\ 10 \end{matrix}\right. & \begin{matrix} 12 \\ 18 \end{matrix} & \left.\begin{matrix} 20 \\ 12 \end{matrix}\right) & \begin{matrix} \leftarrow girls \\ \leftarrow boys \end{matrix} \end{array}$$

If the prize money for three values were respectively Rs 500, Rs200 & Rs300 . Based on the above data, answer the following questions

1	Total award money received by boys a)10000 b)12000 c)12200 d)17400	
2	Total award money received by girls a)12000 b)12200 c)10000 d)17400	
3	Total award money received by the students only for discipline a)14000 b)24000 c)17400 d)12200	
4	How much amount received by the girls for attendance? a)2400 b) 2600 c)2800 d)2000	
5	Who received more amount, boys or girls and how much? a)Girls: 1200 b)Boys:5200 c)Girls:5200 d)Boys: 1200	

17 A manufacturer produces three types of bolts, x, y and z which he sells in two markets. Annual sales (in Rs) are indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

If unit sales prices of x, y and z are Rs.2.50, Rs.1.50 and Rs.1.00 respectively, then answer the following questions using the concept of matrices.

1	Find the total revenue collected from the Market-I. (a) Rs.44000 (b) Rs. 48000 (c) Rs.46000 (d) Rs.53000	
2	Find the total revenue collected from the Market-II. (a) Rs.5100 (b) Rs.53000 (c) Rs.46000 (d) Rs 49000	
3	If the unit costs of the above three commodities are Rs.2.00, Rs.1.00 and 50 paise respectively, then find the gross profit from both the markets. (a) Rs.53000 (b) Rs.46000 (c) Rs.34000 (d) Rs.32000	
4	If matrix A=[aij]2×2 , where aij=1, if i≠j , and aij= 0 if i=j , then A^2 is equal to (a) I (b) A (c) 0 (d) none of these	
5	If A and B are matrices of same order, then (AB' - BA') is a (a) skew-symmetric matrix (b) null matrix (c) symmetric matrix (d) unit matrix	

18 Three car dealers, say A, Band C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan,5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan,5 SUV cars in 2020.

Based on the above information, answer the following questions.

	1	The matrix summarising sales data of 2019 is a) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 30 & 5 \end{bmatrix} \end{array}$ b) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$ **c)** $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 100 & 30 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$ *d)* $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 200 & 50 & 6 \\ 100 & 30 & 5 \\ 300 & 150 & 20 \end{bmatrix} \end{array}$	
	2	The matrix summarizing sales data of 2020 is a) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array}$ b) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 120 & 50 & 10 \\ 100 & 60 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$ **c)** $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 100 & 60 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$ *d)* $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 200 & 50 & 6 \\ 100 & 60 & 5 \\ 300 & 150 & 20 \end{bmatrix} \end{array}$	
	3	The total number of cars sold in two given years, by each dealer, is given by the matrix a) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 190 & 100 & 7 \\ 300 & 80 & 11 \\ 420 & 200 & 30 \end{bmatrix} \end{array}$ b) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 300 & 80 & 11 \\ 190 & 100 & 7 \\ 420 & 200 & 30 \end{bmatrix} \end{array}$ **c)** $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix} \end{array}$ *d) None of these*	
	4	The increase in sales from 2019 to 2020 is given by the matrix a) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 180 & 100 & 10 \\ 10 & 20 & 1 \\ 100 & 20 & 3 \end{bmatrix} \end{array}$ b) $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 10 & 20 & 3 \\ 100 & 20 & 1 \\ 180 & 100 & 10 \end{bmatrix} \end{array}$ **c)** $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 180 & 100 & 10 \\ 100 & 20 & 1 \\ 10 & 20 & 3 \end{bmatrix} \end{array}$ *d)* $\begin{array}{cc} & \begin{array}{ccc} H & S & SUV \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 100 & 20 & 3 \\ 180 & 100 & 10 \\ 10 & 20 & 3 \end{bmatrix} \end{array}$	**1**

5 If each dealer receive profit of Rs.50000 on sale of a Hatchback, Rs.100000 on sale of a Sedan and Rs. 200000 on sale of a SUV (then amount of profit received in the year 2020 by each dealer is given by the matrix **1**

a)$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 30000000 \\ 15000000 \\ 12000000 \end{bmatrix}$ b) $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 12000000 \\ 16200000 \\ 34000000 \end{bmatrix}$ *c)* $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$ *d)* $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 15000000 \\ 30000000 \\ 12000000 \end{bmatrix}$

19 Three people denoted by P1, P2, P3 intend to buy some rolls, buns, cakes and bread. Each of them needs these commodities in differing amounts and can buy them in two shops S1, S2. The individual prices and desired quantities of the commodities are given in the following tables:




Demanded quantity of foodstuff:

	roll	bun	cake	bread
P_1	6	5	3	1
P_2	3	6	2	2
P_3	3	4	3	1

Prices in shops $S1$ and $S2$:

	S_1	**S_2**
roll	**1.50**	**1.00**
bun	**2.00**	**2.50**
cake	**5.00**	**4.50**
bread	**16.00**	**17.00**

1 The amount spent by the person P1 in the shop S1 is

a) 55 **b) 50** **c) 60** **d) 62.5**

2 The amount spent by person P1 in the shop S2 is

a) 49 **b) 45** **c) 48** **d) 51**

3 P2 and P3 are friends and decide to buy all the items from the shop S2. How much will it cost them?

a) 53.5 b) 82 c) 104.5 d) None of the above

4 If the cost of production for S2 to give desired quantities of foodstuff is 113.5. Find the profit earned by S2

a) 38.5 b) 41.5 c) 40 d) None of the above

20 Three schools A, B and C organized an event for collecting funds for helping the COVID effected victims. They sold toys, handmade boxes and notebook packets from the recycled material at a cost of 50, 30 and 40 each. The number of items sold is given below

ITEMS	A	B	C
TOYS	20	30	40
BOXES	50	40	50
NOTEBOOKS	40	20	30



Based on the above information answer the following

Q1. Amount collected by school A is
a) 3500 b) 4100 c) 4500 d) 5100
Q2. Amount collected by school C is
a) 5700 b) 6000 c) 3500 d) 4700
Q3. The total amount raised by all the schools is

a) 12100 b) 12200 c) 12300 d) 12400
Q4. Total amount raised by toys is
a) 4000 b) 4500 c) 5000 d) 5500

21 For what value of x: $(1 \quad 2 \quad 1)\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

22 If A =$\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$,Prove that A+ A^T is symmetric and A – A^T is skew symmetric matrices

23	If f(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that f(x) f(y) = f (x +y).	
24	If A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$	
25	Find x, y, z if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A^T = A^{-1}$	
26	If A = $\begin{bmatrix} 0 & -\tan\frac{\propto}{2} \\ \tan\frac{\propto}{2} & 0 \end{bmatrix}$, and I is the identity matrix of order 2, show that I + A = (I – A) $\begin{bmatrix} cos \propto & -sin \propto \\ sin \propto & cos \propto \end{bmatrix}$	
27	Find the value of a,b,c and d from the equation $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$	
28	If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ A $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find A	
29	Let A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$,C= $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = 0	
30	If A =$\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3	

ANSWERS

QUESTION NO.	CORRECT OPTION
1	1
2	2
3	2
4	2
5	4

6	Prove that $(A A^{I})^{I} = A A^{I}$ and $(A^{I} A)^{I} = A^{I} A$	

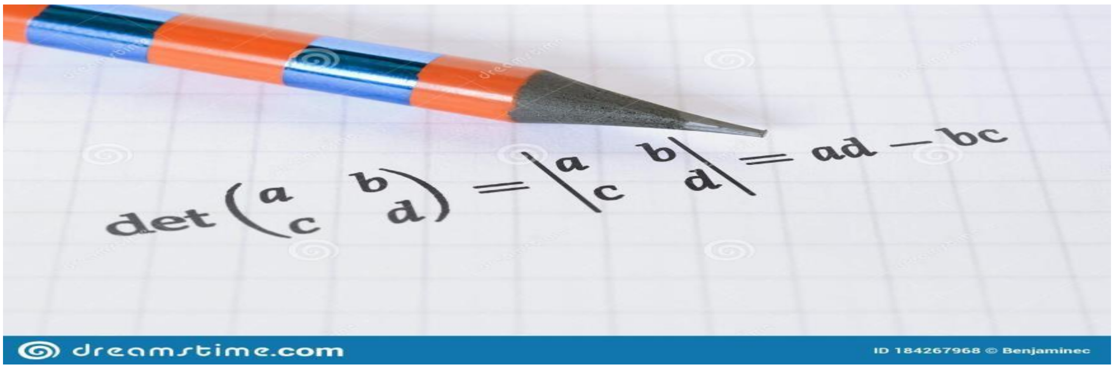
7	$\begin{bmatrix}1 & 0\\2 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\-1 & 3\end{bmatrix}$	
8	A = -2, b = 0 , c = -3 a+b+c = -5	
9	A = $\begin{bmatrix}-2 & 0\\-1 & -3\end{bmatrix}$, B = $\begin{bmatrix}2 & 1\\2 & 2\end{bmatrix}$	
10	Let A = $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ 2a+4b=0, 3a+5b=-3, 2c+4d=10, 3c+5d=3 A = $\begin{bmatrix}-6 & 3\\-19 & 12\end{bmatrix}$	
11	A = $\begin{bmatrix}3 & 3 & \frac{5}{2}\\3 & 1 & \frac{9}{2}\\\frac{5}{2} & \frac{9}{2} & 7\end{bmatrix} + \begin{bmatrix}0 & -1 & \frac{5}{2}\\1 & 0 & -\frac{3}{2}\\-\frac{5}{2} & \frac{3}{2} & 0\end{bmatrix}$	
12	f(A) =A^2 -2A-3I = $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	
13	K= $\begin{bmatrix}-7 & 0\\0 & -7\end{bmatrix}$	
14	Prove that $(AB^I - BA^I)^I = -(AB^I - BA^I)$, Using Properties	
15	$(A-I)^3 = A^3 -3A^2 I +3AI^2 -I^3$ $(A+I)^3 = A^3 +3A^2 I +3AI^2 +I^3$	

	Q1	Q2	Q3	Q4	Q5
16	c	d	a	a	c
17	c	b	d	a	a
18	b	a	c	c	c
19	b	a	c	c	
20	b	d	c	b	

21	**X = -1**	

22	Let P = A + A^T = $\begin{bmatrix} 4 & 7 \\ 1 & 10 \end{bmatrix}$, and Q = A - A^T = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, prove that P^T =P and Q^T = -Q	
23	F (x + y) = $\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
24	**A^2=**$\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$, A^3==$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$	
25	AA^T =I $4y^2+z^2=1$, $2y^2-z^2=0$, $x^2+y^2+z^2=1$ **X=**$\pm\frac{1}{\sqrt{2}}$ **, y = =**$\pm\frac{1}{\sqrt{6}}$ **, z = =**$\pm\frac{1}{\sqrt{3}}$ **,**	
26	**I + A =** $\begin{bmatrix} 1 & -\tan\frac{\propto}{2} \\ \tan\frac{\propto}{2} & 1 \end{bmatrix}$, **I - A =** $\begin{bmatrix} 1 & \tan\frac{\propto}{2} \\ -\tan\frac{\propto}{2} & 1 \end{bmatrix}$,	
27	a=1 , b = 2, c = 3, d =4	
28	**A =** $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	
29	**D =** $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$	
30	**A^2 -5A -14I =** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ A^3 = $\begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$	

CHAPTER 4
DETERMINANTS



CONCEPT MAPPING

DETERMINANT OF A SQUARE MATRIX

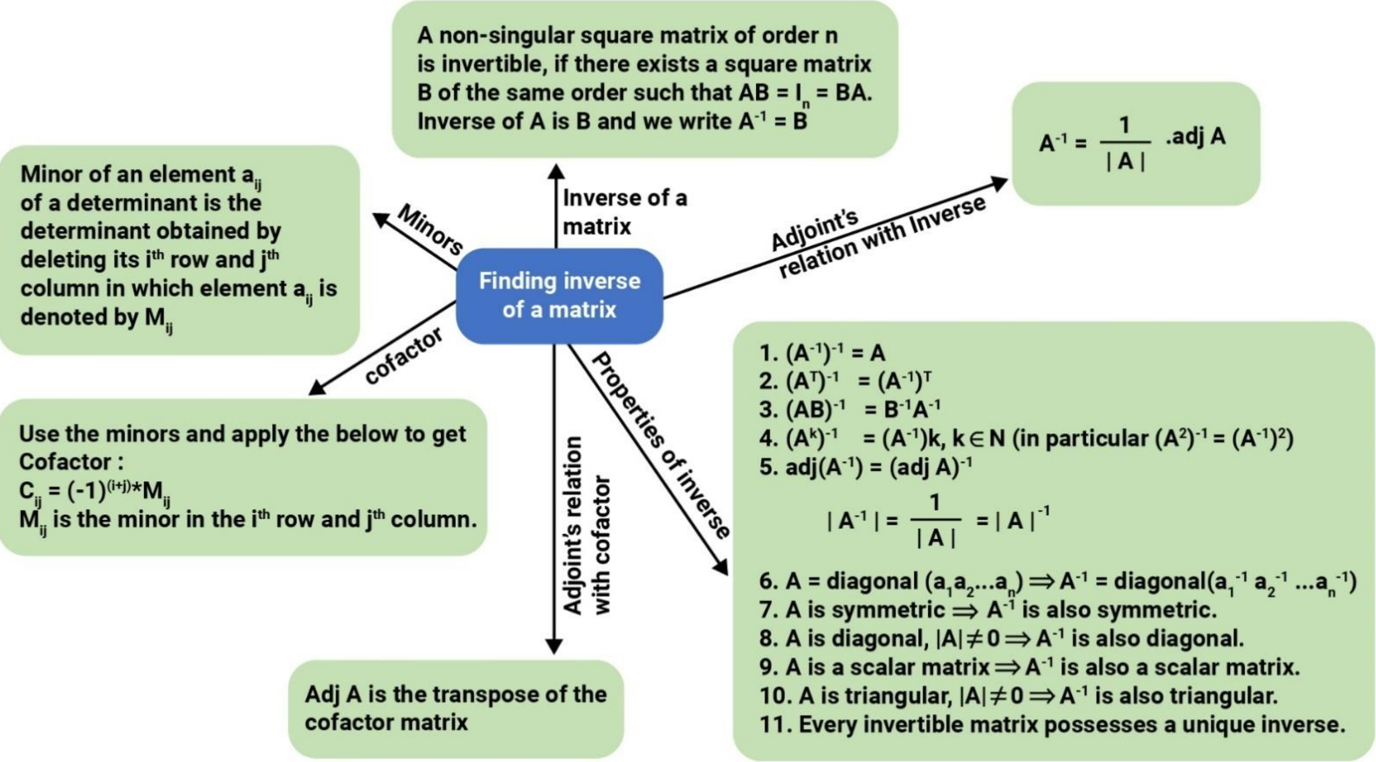
MINOR AND CO-FACTOR

AREA OF TRIANGLE

ADJOINT OF A MATRIX

INVERSE OF A MATRIX

SOLUTION OF EQUATION BY MATRIX METHOD



A non-singular square matrix of order n is invertible, if there exists a square matrix B of the same order such that $AB = I_n = BA$. Inverse of A is B and we write $A^{-1} = B$

$$A^{-1} = \frac{1}{|A|} .\text{adj } A$$

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} is denoted by M_{ij}

Use the minors and apply the below to get Cofactor :
$C_{ij} = (-1)^{(i+j)} * M_{ij}$
M_{ij} is the minor in the ith row and jth column.

Adj A is the transpose of the cofactor matrix

1. $(A^{-1})^{-1} = A$
2. $(A^T)^{-1} = (A^{-1})^T$
3. $(AB)^{-1} = B^{-1}A^{-1}$
4. $(A^k)^{-1} = (A^{-1})k, k \in N$ (in particular $(A^2)^{-1} = (A^{-1})^2$)
5. $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$

$$|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$$

6. $A = \text{diagonal } (a_1a_2...a_n) \Rightarrow A^{-1} = \text{diagonal}(a_1^{-1} a_2^{-1} ...a_n^{-1})$
7. A is symmetric ⇒ A^{-1} is also symmetric.
8. A is diagonal, $|A| \neq 0 \Rightarrow A^{-1}$ is also diagonal.
9. A is a scalar matrix ⇒ A^{-1} is also a scalar matrix.
10. A is triangular, $|A| \neq 0 \Rightarrow A^{-1}$ is also triangular.
11. Every invertible matrix possesses a unique inverse.

GIST OF THE LESSON

To every square matrix we can assign a number called determinant

Value of determinant of a matrix of order 1

If A= [aij]. Then Det .A= IAI= $|aij|$ = aij

Value of determinant of a matrix of order 2

If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then IAI = $a_{11}a_{22}$ - $a_{21}a_{12}$

Value of determinant of a matrix of order 3

. If A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then IAI = $a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

If A= [aij]. Is a square matrix of order n ,then IAI =$|A^T|$

If A and B are square matrix of order n, then$|AB|=|A||B|$

Let A be a square matrix of order n × n, then | kA| is equal to k^n IAI

The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The area of the triangle formed by three collinear points is zero.

Equation of line joining the points $(x_1, y_1)$and $(x_2, y_2)$is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Cofactor of an element a_{ij} ,denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$,where M_{ij} is minor of a_{ij}

Δ = a11A11+a12A12+a13A13.where Aij are cofactors of aij.

If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero

$\Delta = a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

The adjoint of a square matrix A = $[a_{ij}]$.n × n is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij} . Adjoint of the matrix A is denoted by adj A.

If A be a square matrix of order n, then $|adjA|=|A|^{n-1}$.

If A be a square matrix of order n, then $|A\ adjA|=|A|^n$.

For any square matrix A, A(Adj.A)=(Adj.A)A=$|A|$I

If A be a square matrix of order n, then $|adj(adjA)|= \ |A|^{(n-1)^2}$

If A and B are square matrices of the same order, then adj(AB)=adjB.adj A

If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then adj.A = $\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$ Note :to find adjA interchange diagonal elements and change the sign of non – diagonal elements.

If A is a singular matrix, then $|A|$=0

A square matrix A is said to be non-singular if $|A| \neq 0$

If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order

A square matrix A is invertible if and only if A is nonsingular matrix ($|A| \neq 0$)

$A^{-1} = \frac{1}{|A|}\ (adjA)$

If A is an invertible matrix, then $|A| \neq 0$ and$(A^{-1})^T = (A^T)^{-1}$

If A is a non-singular matrix $|(kA)^{-1}| = \frac{1}{k\ |A|}$

$(AB)^{-1} = B^{-1}A^{-1}$

<u>Solution of system of linear equations using inverse of a matrix</u>

Consider the system of equations

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ AX = B , $X = A^{-1}$ B

Where $A^{-1} = \frac{1}{|A|}\ (adjA)$

MULTIPLE CHOICE QUESTIONS

1	The area of a triangle with vertices (2 , - 6),(5 , 4)and(K , 4) is 35 sq. units then k is a)12 b) – 2 c) -12 , -2 d)12 , - 2

2	If A= $\begin{bmatrix}5&10&3\\-2&-4&6\\-1&-2&b\end{bmatrix}$ is a singular matrix then value of b is A)-3 b) 3 c) 0 d) arbitrary
3	**Themaximum valueof** $\begin{vmatrix}1&1&1\\1&1+\sin\theta&1\\1&1&1+\sin\theta\end{vmatrix}$ A) -2 b) 1 c) -1 d) 2
4	Let A is a non – singular matrix of order 3 x 3 then $\|A\;(adj\;A)\|$ is equal to A)$\|A\|$b) $\|A\|^2$ c) $\|A\|^3$ d) 3$\|A\|$
5	If A and B are invertible matrices of order 3,$\|A\|$=2 and $\|(AB)^{-1}\|$= - 1/ 6 , Find $\|B\|$ A) – 1/3 b) 3 c) -1/12 d) -3
SHORT ANSWER QUESTIONS	
6.	If A = $\begin{bmatrix}2&\beta&-4\\0&2&5\\1&1&3\end{bmatrix}$ find value of β for which A^{-1} exist ?
7	If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix}2&-3&5\\6&0&4\\1&5&-7\end{vmatrix}$, find the value of a_{32} A_{32}
8.	If A is a square matrix of order 3 x3 with $\|A\| = 9$, then write the value of $\|\,2\,.\,adjA\|$
9.	For the matrix A = $\begin{bmatrix}3&2\\1&1\end{bmatrix}$,find the numbers a and b such that $A^2 + a\,A + b\,I =$ **O**
10	There are two values of a which makes determinant, $\Delta = \begin{vmatrix}1&-2&5\\2&a&-1\\0&4&2a\end{vmatrix} = 86$ Then find sum of these numbers.
11	If A = $\begin{bmatrix}1&3\\2&1\end{bmatrix}$, find the value of $\|\;A^2\; - 2\,A\;\|$
12	Find k if the matrix $\begin{bmatrix}1&k&3\\1&3&3\\2&4&4\end{bmatrix}$ is the adjoint of a 3x 3 matrix A and $\|A\| = 4$
13	$\begin{vmatrix}2x&5\\8&x\end{vmatrix} = \begin{vmatrix}6&5\\8&3\end{vmatrix}$, then find x.

14	Show that if the determinant $\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$ then sin θ = 0 or sin θ = 1/2 .
15	Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 7\,5^0 & \cos 75^0 \end{vmatrix}$
16	If A = $[a_{ij}]$ is a matrix of order 2 x2 , such that $\|A\|$ = - 15 and A_{ij} represents the cofactor of a_{ij} then find $a_{21}A_{21} + a_{22}A_{22}$
17	If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$ then find the value of x
18	If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then find the value of k if $\|2A\| = k\|A\|$
19	Show that the points (a, b+c) , (b , c+a) , and (c, a+b) are collinear
20	Find the interval in which Det (A) lies if $A = = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$
LONG ANSWER QUESTIONS	
21	If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$. Hence find A^{-1}
22	If $A = \begin{bmatrix} 12 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A^T)^{-1}$
23	Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = \mathbf{O}$ where I is 2 x 2 identity matrix and **O** is 2 x 2 zero matrix. Using this equation , find A^{-1}.
24	Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ compute A^{-1} and show that $2A^{-1} = 9I - A$
25	If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$
26	If x = – 4 is a root of $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$ then find the other two roots
27	Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$ then find $\lim_{t \to 0} \frac{f(t)}{t^2}$
28	Find the equation of the line joining A (1 , 3) and B (0, 0) using determinants and find k if D (k , 0) is a point such that area of triangle ABD is 3 sq. units.
29	If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, verify $A(\text{adj } A) = \|A\| I$ and find A^{-1}

30 Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$X - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$

31 Find x , y and z if $A = \begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A^T = A^{-1}$

32 A typist charges Rs 145 for typing 10 English and 3 Hindi pages , while charges for typing 3 English and 10 Hindi pages are Rs 180 .Using matrices , find the charges of typing one English and one Hindi page separately.

33 If $A^{-1} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B = \begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ find $(AB)^{-1}$

34 Find A^{-1} , if $A = \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$. Also show that $A^{-1} = \frac{A^2 - 3I}{2}$

35 The monthly income of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5:7 . if each saves rs 15,000 per month , find their monthly incomes using matrix method

CASE STUDY QUESTIONS

36 Two schools A and B decided to award prizes to their students for three values honesty (x) ,punctuality (y) and obedience (z) .School A decided to award a total of Rs 11000 for the three values to 5 , 4 and 3 students respectively while school B decided to award Rs 10700 for the three values to 4 , 3 and 5 students respectively . If all the three prizes together amount to Rs 2700 , based on the information given answer the following questions

1. Form the equations in terms of x , y and z
2. Is it possible to solve the system of equations using matrix method
3. Find award prize for each of the three values

37 A mixture is to be made of three foods A , B , C . The three foods A ,B, C contain nutrients P , Q , R shown below .

Food	Grams per kg of nutrient		
	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

1. Form linear equation representing the data if the mixture will have 8 kg of P, 5 kg of Q and 7 kg of R
2. How to form a mixture which will have 8 kg of P, 5 kg of Q and 7 kg of R.

38

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2

1. Based on the information given above, form equations in terms of x and y
2. Write down matrix equation represented by the given information
3. Find the value of x (length of rectangular field)
4. Find the value of y (breadth of rectangular field)
5. How much is the area of rectangular field?

39

Three shopkeepers Salim, Vijay and Venket are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Salim, Vijay and Venket are using (20,30,40), (30,40,20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Salim, Vijay and Venket spent Rs.250, Rs.270 and Rs.200 on these carry bags respectively.

Using the concept of matrices and determinants, answer the following questions.

(i) What is the cost of one polythene bag?

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs.5

(ii) What is the cost of one handmade bag?

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5

(iii) What is the cost of one newspaper envelope

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5

(iv) Keeping in mind the social conditions, which shopkeeper is better?

(a) Salim (b) Vijay (c) Venket (d) None of these

(v) Keeping in mind the environmental conditions, which shopkeeper is better?

	(a) Salim (b) Vijay (c) Venket (d) None of these
40	A trust invested some money in two types of bonds . The first bond pays 10 % interest and second bond pays 12 % interest .The trust received Rs 2800as interest . However , if trust had interchanged money in bonds ,they would have got Rs 100 less as interest . 1. Write down linear equations based on information given above 2. Write down matrix equation represented by the given information 3.Find the amount invested in first bond 4. Find the amount invested in first bond 5. Find the total amount invested by the trust

ANSWERS

MULTIPLE CHOICE QUESTIONS

1) d)12 , - 2	2) d) arbitrary	3) b) 1	4) c) $\mid A \mid^3$	5) d) -3

SHORT ANSWER QUESTIONS

6) $\beta \neq -2$	7) 110	8) 8 x 9 ^{2} = 648	9) a = -4 , b = 1

10) $\Delta = 86$

$\rightarrow a^2 + 4a - 21 = 0$

$a = -7, 3$

Sum = -4

11) 25

12) $|\, adj\ A \,| = |\, A \,|^{n-1}$

$2k - 6 = 4^2$

K = 11

13) $x = \pm 3$

14) $2 - 2\cos 2\theta - \sin 3\theta = 0$

$2 - 2(1 - 2\sin^2\theta) - (3\sin\theta - 4\sin^3\theta) = 0$

$\sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\sin\theta = 0 \text{ or } \sin\theta = 1/2$

15) $\cos 90^0 = 0$

16) $|\ A\ | = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Expanding along R2

$a_{21} A_{21} + a_{22} A_{22} = -15$

17) x = 2	18) k = 4

19) Given points are collinear as $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0$	20) Det (A) ∈ [2 ,4]

LONG ANSWER QUESTIONS

21) x = 8 , y = 8 ,

$A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

22) $|A^T| = 1$

$$(A^{T})^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

23) $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

24) $|A| = 2$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$9I - A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= 2A^{-1}$

25) $(AB)^{-1} = B^{-1}A^{-1} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

26) $\Delta = 0$

$\rightarrow x^3 - 13x + 12 = 0$

x = 1 and x = 3 are other two roots.

27) $f(t) = -t^2 \cos t + t \sin t$

$\lim_{t \to 0} \frac{f(t)}{t^2} = 0$

28) Equation of line AB is $y = 3x$

Area of triangle = 3 sq units

$\rightarrow k = \pm 2$

29) $|A| = 1$

$\text{Adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A|\, I$

$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

30) Product $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$X = 0,\ y = 5,\ z = 3$

31) $A^T = A^{-1}$

$A A^T = A A^{-1}$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing corresponding elements

$2y^2 - z^2 = 0$

$4y^2 + z^2 = 1$

$x^2 + y^2 + z^2 = 1$

$x = \pm\frac{1}{\sqrt{2}}$, $y = \pm\frac{1}{\sqrt{6}}$, $z = \pm\frac{1}{\sqrt{3}}$

32) Let charges of typing 1 English page be Rs x and of 1 Hindi page be Rs y

$10x + 3y = 145$

$3x + 10y = 180$

$|A| = 91$ and $A^{-1} = \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$

X = 10 , y = 15

33) $|B| = 1$

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} , \quad (AB)^{-1} = B^{-1} A^{-1} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

,

34) $|A| = 2$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\frac{A^2 - 3I}{2} = \frac{1}{2} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \frac{3}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = A^{-1}$$

35) 3x- 5y = 15000

4x – 7y = 15000

$|A| = -1$

$$A^{-1} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

X = 30000

Y = 15000

Monthly income of Aryan = 3 x 30000= Rs 90000

Monthly income of Babban = 4 x 30000= Rs 1,20,000

CASE STUDY QUESTIONS.

36) 1 . 5x+4y+3z = 11000 , 4x + 3y +5z = 10700 , x + y + z = 2700

2. yes as $|A| = -3$

3. x = 1000 , y = 900 , z = 800

37) Let the food needed be x kg of A , y kg of B and z kg of C

x+3y+4z =8 , 2x +y+2z = 5 , 5x +y+z = 7

$|A| = 11$

x = 1 , y =1 , z = 1

The mixture is formed by mixing 1 kg of each of food A, B , C

38) 1. (x-50)(y+50)=xy

→ x – y = 50

(x- 10)(y – 20) = xy – 5300

→ 2x+y=550

2. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

3. x= 200

4. y = 150

5 Area =200 x 150 = 30, 000 sq .m

39) 20x + 30y + 40z = 250, 30x + 40y + 20z = 270

40x + 20y + 30z .=. 200

$|A| =$ -27000

By matrix method cost of a polythene bag, a handmade bag and a newspaper envelope is Rs.1, Rs.5 and Rs.2 respectively.

(i) (a)

(ii) (d)

(iii) (b)

(iv) (b) : Vijay investing most of the money on hand-rnade bags.

(v) (a) : Salim investing less amount of money on polythene bags.

40) 1. 10x +12y = 280000

12x+10y =270000

2. $\begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ = $\begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$

3 . x= 10000

4. y = 15000

ANSWERS

MULTIPLE CHOICE QUESTIONS

1) d)12 , - 2	2) d) arbitrary	**3)** b) 1	4) c) $\mid A \mid^3$	**5)** d) -3

SHORT ANSWER TYPE QUESTIONS

6) $\beta \neq -2$	**7) 110**	**8)** 8×9^2 **= 648**	**9) a = -4 , b = 1**

10) $\Delta = 86$

$\rightarrow a^2 + 4a - 21 = 0$

$a = -7, 3$

Sum = -4

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12) $|\ adj\ A\ | = |\ A\ |^{n-1}$

2k – 6 = 4^2

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$\sin\theta = 0$ or $\sin\theta = 1/2$

15) $\cos 90^0 = 0$

16) $|\ A\ | = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Expanding along R₂

$a_{21}A_{21} + a_{22}A_{22}$ **= - 15**

17) x = 2	**18) k = 4**

19) Given points are collinear as $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0$	**20) Det (A) ∈ [2 ,4]**

LONG ANSWER TYPE QUESTIONS

21) x = 8 , y = 8 ,

$A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

22) $|A^T| = 1$

$(A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

23) $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

24) $|A| = 2$

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25) $(AB)^{-1} = B^{-1} A^{-1} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

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28) Equation of line AB is y = 3 x

Area of triangle = 3 sq units

$\rightarrow k = \pm 2$

29) $|A| = 1$

$$\text{Adj A} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad , \text{A (adj A)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A|\ I$$

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

30) Product = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$X = 0\ ,\ y = 5\ ,\ z = 3$

31) $A^T = A^{-1}$

$A A^T = A A^{-1}$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$2y^2 - z^2 = 0$

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,

34) $|A| = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\frac{A^2 - 3I}{2} = \frac{1}{2}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = A^{-1}$$

35) $3x - 5y = 15000$

$4x - 7y = 15000$

$|A| = -1$

$$A^{-1} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

$X = 30000$

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CASE STUDY TYPE QUESTIONS.

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2. yes as $|A| = -3$

3. $x = 1000$, $y = 900$, $z = 800$

37) Let the food needed be x kg of A , y kg of B and z kg of C

$x+3y+4z = 8$, $2x + y + 2z = 5$, $5x + y + z = 7$

$|A| = 11$

$x = 1 , y = 1 , z = 1$

The mixture is formed by mixing 1 kg of each of food A, B , C

38) 1. $(x-50)(y+50) = xy$

$\rightarrow x - y = 50$

$(x-10)(y-20) = xy - 5300$

$\rightarrow 2x + y = 550$

2. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

3. x= 200

4. y = 150

5 Area =200 x 150 = 30, 000 sq .m

---

39) 20x + 30y + 40z = 250, 30x + 40y + 20z = 270
40x + 20y + 30z .=. 200
$|A| =$ -27000

By matrix method cost of a polythene bag, a handmade bag and a newspaper envelope is Rs.1, Rs.5 and Rs.2 respectively.

(i) (a)
(ii) (d)
(iii) (b)
(iv) (b) : Vijay investing most of the money on hand-rnade bags.
(v) (a) : Salim investing less amount of money on polythene bags.

---

40) 1. 10x +12y = 280000
12x+10y =270000

3. $\begin{bmatrix}10 & 12\\12 & 10\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}280000\\270000\end{bmatrix}$

4 . x= 10000
5. y = 15000

CONTINUITY AND DIFFERENTIABILITY.

GIST OF THE LESSON FOR QUICK REVISION

Continuous Function

A real valued function f is said to be continuous, if it is continuous at every point in the domain of f.

Continuity of a function at a point

Suppose f is a real valued function on a subset of real numbers and let c be a point in the domain of f, then f is continuous at x=c, if $\lim_{x\to c} f(x) = f(c)$

Ie., $\lim_{x\to c+} f(x) = \lim_{x\to c-} f(x) = f(c)$

Then f(x) is continuous at x=c . Otherwise f(x) is discontinuous at x=c

Graphically, a function f(x) is said to be continuous at a point if the graph of the function has no break either on the left or the right in the neighbourhood of the point.

Some basic continuous functions:

1. Every constant function is continuous.
2. Every identity function is continuous.
3. Rational functions are always continuous
4. Every polynomial function is continuous.
5. Modulus function f(x)= |x| is continuous.
6. All trigonometric functions are continuous in their domain.
7. e^x, logx continuous in their domain.

Algebra of continuous function

Theorem 1 :

Let f and g be two real functions, continuous at a real number c, then

1. (f + g) is continuous at x=c
2. (f-g) is continuous at x=c
3. fg is continuous at x=c
4. f/g is continuous at x=c provided g(c) ≠0

Differentiability

A real valued function f, is said to be differentiable at x=c in its domain, if its left hand and right hand derivatives at x=c exists and are equal.

At x=a, right hand derivative,

Rf'(a)=$\lim_{x\to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$ and left hand derivative, Lf'(a) = $\lim_{x\to 0}\left(\frac{f(a-h)-f(a)}{-h}\right)$

Thus f(x) is differentiable at x=a , if Rf'(a) = Lf'(a)

Otherwise, f(x) is not differentiable at x=a

Derivatives of some standard functions:

1. $\frac{d}{dx}$(constant) =0
2. $\frac{d}{dx}(x^n)$ =nx^{n-1}
3. $\frac{d}{dx}$(sin x) = cos x
4. $\frac{d}{dx}$(cos x) = -sin x
5. $\frac{d}{dx}$(tan x) = sec^2x
6. $\frac{d}{dx}$(cosec x) = $-cosec\ x \cot x$
7. $\frac{d}{dx}$(sec x) = sec x tan x
8. $\frac{d}{dx}$(cot x) = $-cosec^2x$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = a^x$ log a, a> 0
11. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$, x> 0
12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$, a>0 , a ≠1

Algebra of derivatives

1. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
2. $\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ (product rule)
3. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$ (quotient rule) where u and v are functions of x
4. $\frac{d(ku)}{dx} = k.\frac{du}{dx}$ where k is a constant

Chain Rule

Example 1:

If y = $\sin(x^2)$, then $\frac{dy}{dx} = \cos(x^2).\frac{d}{dx}(x^2) = \cos(x^2).2x = 2x\cos(x^2)$.

Example 2:

If y= tan(2x+3) , then $\frac{dy}{dx} = sec^2(2x+3).\frac{d}{dx}(2x+3)$

$$= sec^2(2x+3).2 = 2sec^2(2x+3).$$

Example 3 :

If y= $\sin(\cos(x^2))$, then $\frac{dy}{dx} = \cos(\cos(x^2)).\frac{d(\cos(x^2))}{dx}$

$$= \cos(\cos(x^2)).-\sin(x^2).\frac{d}{dx}(x^2)$$

$$= \cos(\cos(x^2)).-\sin(x^2).2x$$

$$= -2x.\sin(x^2)\cos(\cos(x^2))$$

Derivative of implicit function

Let f(x,y) = 0 be an implicit function of x, then, to find $\frac{dy}{dx}$, first differentiate both sides of equation w.r.t x and then take all terms involving $\frac{dy}{dx}$ to LHS and remaining terms to RHS , then find $\frac{dy}{dx}$.

Derivatives of inverse trigonometric functions

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, -1 < x< 1
2. $\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$, -1 < x< 1
3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$
5. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x.\sqrt{x^2-1}}$, |x| > 1
6. $\frac{d}{dx}(\text{cosec}^{-1}x) = \frac{-1}{x.\sqrt{x^2-1}}$, |x| > 1

Derivative of a function w.r.t another function

Let u = f(x) and v= g(x) be two given functions. Differentiate both functions w.r.t x separately and substitute in the following formula.

$$\frac{du}{dv} = \frac{du}{dx} \div \frac{dv}{dx}$$

Derivative of logarithmic functions

Suppose, given function is of the form $u(x)^{v(x)}$

In such cases, take logarithm on both sides and use properties of logarithm to simplify it. Then differentiate it.

Derivatives of parametric functions

If x= f(t) and y = g(t), then

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

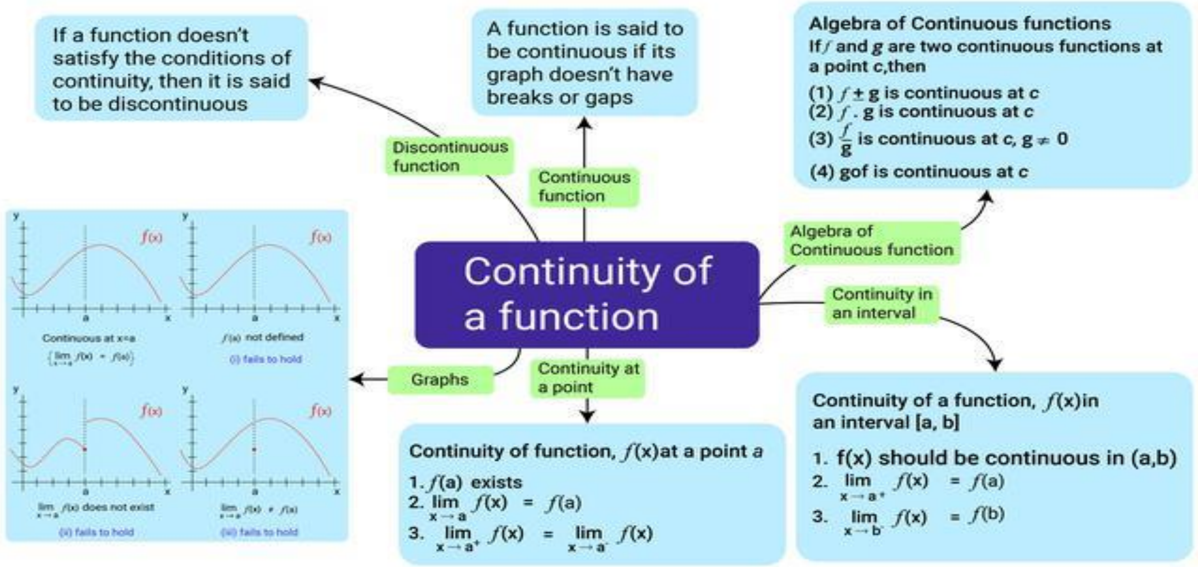
Second order derivative

Let y=f(x) be a given function, then $\frac{dy}{dx}$ is called first derivative of y.

$\frac{d}{dx}(\frac{dy}{dx})$ is called the second order derivative of y w.r.t x and is denoted by

$\frac{d^2y}{dx^2}$ or y'' or y_2

MIND MAPPING



Continuity of a function

Discontinuous function: If a function doesn't satisfy the conditions of continuity, then it is said to be discontinuous

Continuous function: A function is said to be continuous if its graph doesn't have breaks or gaps

Algebra of Continuous function:

Algebra of Continuous functions
If f and g are two continuous functions at a point c,then
(1) $f \pm g$ is continuous at c
(2) $f \cdot g$ is continuous at c
(3) $\frac{f}{g}$ is continuous at c, $g \neq 0$
(4) gof is continuous at c

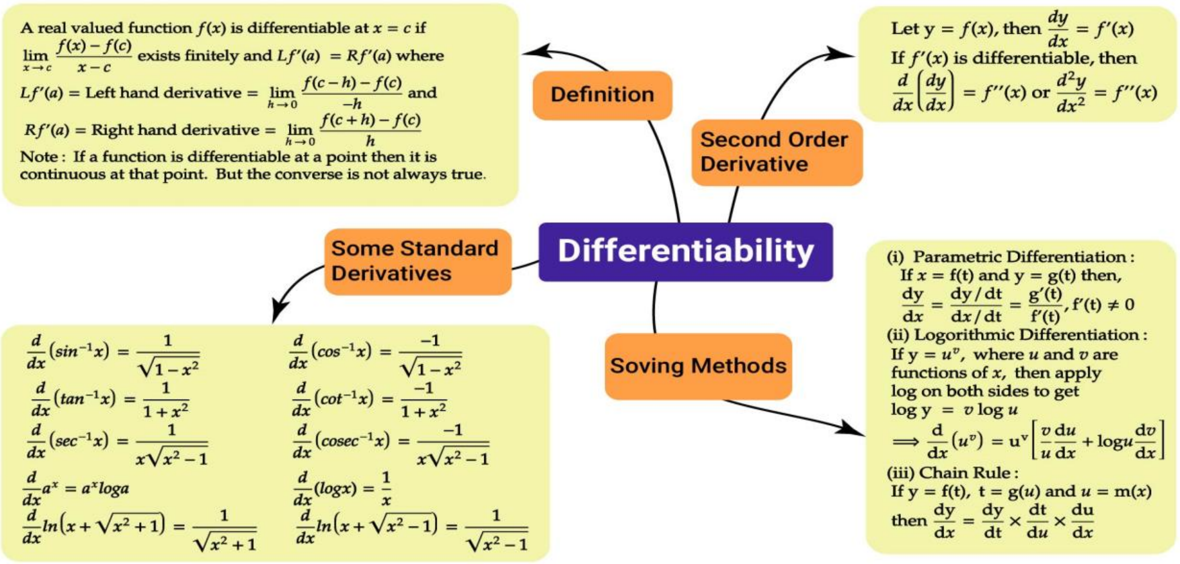
Graphs

Continuity at a point:

Continuity of function, $f(x)$at a point a
1. $f(a)$ exists
2. $\lim_{x\to a} f(x) = f(a)$
3. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

Continuity in an interval:

Continuity of a function, $f(x)$in an interval [a, b]
1. f(x) should be continuous in (a,b)
2. $\lim_{x\to a^+} f(x) = f(a)$
3. $\lim_{x\to b^-} f(x) = f(b)$



Differentiability

Definition:

A real valued function $f(x)$ is differentiable at $x = c$ if $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ exists finitely and $Lf'(a) = Rf'(a)$ where
$Lf'(a)$ = Left hand derivative = $\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$ and
$Rf'(a)$ = Right hand derivative = $\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$
Note : If a function is differentiable at a point then it is continuous at that point. But the converse is not always true.

Second Order Derivative:

Let $y = f(x)$, then $\frac{dy}{dx} = f'(x)$
If $f'(x)$ is differentiable, then $\frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x)$ or $\frac{d^2y}{dx^2} = f''(x)$

Some Standard Derivatives:

$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(cot^{-1}x) = \frac{-1}{1+x^2}$
$\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(cosec^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}a^x = a^x loga$
$\frac{d}{dx}(logx) = \frac{1}{x}$
$\frac{d}{dx}ln\left(x+\sqrt{x^2+1}\right) = \frac{1}{\sqrt{x^2+1}}$
$\frac{d}{dx}ln\left(x+\sqrt{x^2-1}\right) = \frac{1}{\sqrt{x^2-1}}$

Soving Methods:

(i) Parametric Differentiation :
If $x = f(t)$ and $y = g(t)$ then,
$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}, f'(t) \neq 0$
(ii) Logorithmic Differentiation :
If $y = u^v$, where u and v are functions of x, then apply log on both sides to get
$\log y = v \log u$
$\Longrightarrow \frac{d}{dx}(u^v) = u^v\left[\frac{v}{u}\frac{du}{dx} + log u\frac{dv}{dx}\right]$
(iii) Chain Rule :
If $y = f(t)$, $t = g(u)$ and $u = m(x)$
then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dx}$

TYPES OF QUESTIONS

I.MCQ

1 If $f(x) = \begin{cases} 3x-5, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at x=5, then k is

a.7/2 b.2/7 c.-7/2 d.-2/7

2 The function f(x) =[x] is continuous at

a.4 b.-2 c.1 d.1.5

3 If x=t^2 and y=t^3 then $\frac{d^2y}{dx^2}$ is equal to

a. $\frac{3}{2}$ b. $\frac{3}{4t}$ c. $\frac{3}{2t}$ d. $\frac{3t}{2}$

4 Derivative of x^2 with respect to x^3 is

a. $\frac{1}{x}$ b. $\frac{2}{3x}$ c. $\frac{2}{3}$ d. $\frac{3x}{2}$

5 For the curve √x+√y = 1, $\frac{dy}{dx}$ at (1/4,1/4) is

a.1 b.1/2 c.-1 d. none of these

II.SHORT ANSWER QUESTIONS

6 Check whether the function $f(x)=\begin{cases}3x+5, & x\geq 2\\ x^2, & x<2\end{cases}$ is continuous at x=2

7 Show that the function $f(x)=\begin{cases}\frac{x^2-x-6}{x-3}, & x\neq 3\\ 5, & x=3\end{cases}$ is continuous at x=3

8 If the function $f(x)=\begin{cases}3ax+b, & x>1\\ 11 & x=1\\ 5ax-2b & x<1\end{cases}$ is continuous at x=1, then find a and b

9 Prove that $f(x)=\begin{cases}1+x, & x\leq 2\\ 5-x, & x>2\end{cases}$ is not differentiable at x=2

10 Find the value of k for which

$f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & -1\leq x<0\\ \frac{2x+1}{x-1}, & 0\leq x\leq 1\end{cases}$ is continuous at x=0

11 Discuss the continuity of the function at x=1/2 where the function

$$f(x)=\begin{cases}\frac{1}{2}+x, & 0\leq x<1/2\\ 1 & x=1/2\\ \frac{3}{2}+x & 1/2<x\leq 1\end{cases}$$

12 If $x^2 + 2xy + y^3 = 42$, find $\frac{dy}{dx}$

13 If siny =x sin(a+y). prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

14 If $y=e^{3x}\sin 4x2^x$, find $\frac{dy}{dx}$

15 If $x^y = y^x$, find $\frac{dy}{dx}$

16 Differentiate $\tan^{-1}(\frac{2^{x+1}}{1-4^x})$

17 If $y = \tan^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone

18 Differentiate log $(1+x^2)$with respect to $\tan^{-1}x$

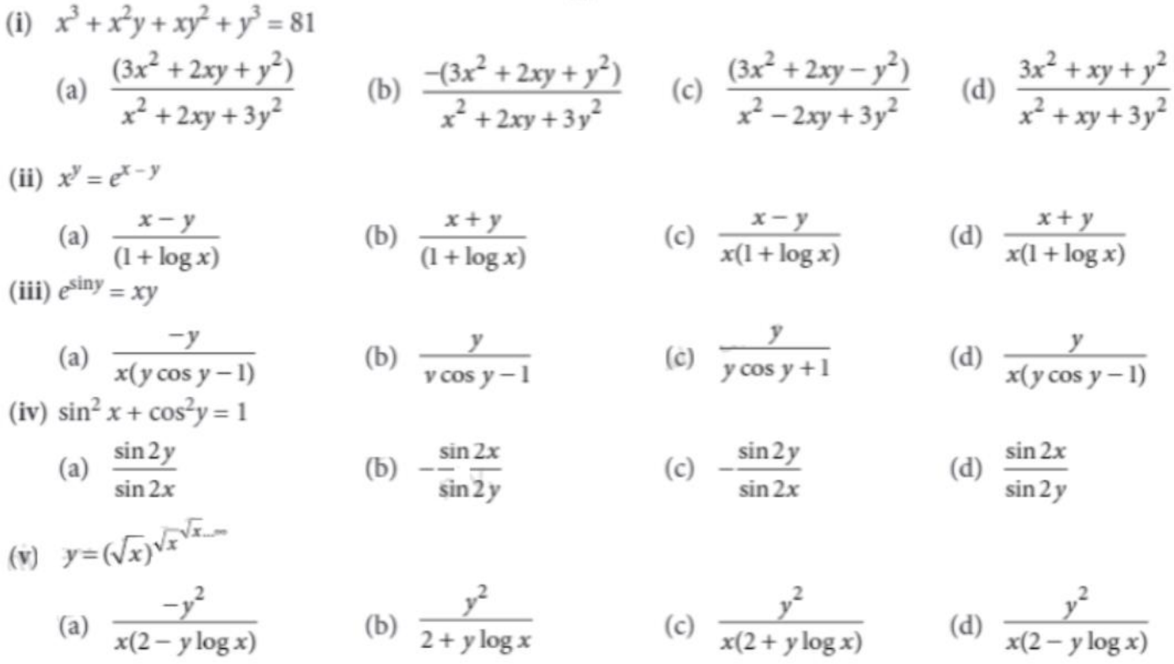
19 If $y=ax^{n+1} + bx^{-n}$ and $x^2\frac{d^2y}{dx^2}=\lambda y$, then find λ

20 Find $\frac{dy}{dx}$, when $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$ and $x = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

III.CASE STUDY QUESTIONS

21 If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation with respect x, remembering that a term in y is first differentiated w.r.t y and then multiplied by $\frac{dy}{dx}$

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions



(i) $x^3 + x^2y + xy^2 + y^3 = 81$

(a) $\frac{(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$ (b) $\frac{-(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$ (c) $\frac{(3x^2 + 2xy - y^2)}{x^2 - 2xy + 3y^2}$ (d) $\frac{3x^2 + xy + y^2}{x^2 + xy + 3y^2}$

(ii) $x^y = e^{x-y}$

(a) $\frac{x-y}{(1+\log x)}$ (b) $\frac{x+y}{(1+\log x)}$ (c) $\frac{x-y}{x(1+\log x)}$ (d) $\frac{x+y}{x(1+\log x)}$

(iii) $e^{\sin y} = xy$

(a) $\frac{-y}{x(y\cos y - 1)}$ (b) $\frac{y}{v\cos y - 1}$ (c) $\frac{y}{y\cos y + 1}$ (d) $\frac{y}{x(y\cos y - 1)}$

(iv) $\sin^2 x + \cos^2 y = 1$

(a) $\frac{\sin 2y}{\sin 2x}$ (b) $-\frac{\sin 2x}{\sin 2y}$ (c) $-\frac{\sin 2y}{\sin 2x}$ (d) $\frac{\sin 2x}{\sin 2y}$

(v) $y = (\sqrt{x})^{\sqrt{x}^{\sqrt{x}\dots\infty}}$

(a) $\frac{-y^2}{x(2 - y\log x)}$ (b) $\frac{y^2}{2 + y\log x}$ (c) $\frac{y^2}{x(2 + y\log x)}$ (d) $\frac{y^2}{x(2 - y\log x)}$

22 If y = f(u)is a differentiable function of u and u = g(x is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. This rule is known as CHAIN RULE.

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions

1.cos√x

a. $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ b. $\frac{\sin\sqrt{x}}{2\sqrt{x}}$ c.sin√x d. -sin√x

2. $7^{x+\frac{1}{x}}$

a. $\left(\frac{x^2-1}{x^2}\right)7^{x+\frac{1}{x}}\log 7$ b. $\left(\frac{x^2+1}{x^2}\right)7^{x+\frac{1}{x}}\log 7$ c. $\left(\frac{x^2-1}{x^2}\right)7^{x-\frac{1}{x}}\log 7$

d. $\left(\frac{x^2+1}{x^2}\right)7^{x-\frac{1}{x}}\log 7$

3. $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

a. $\dfrac{1}{2}\sec^2\dfrac{x}{2}$ b. -$\dfrac{1}{2}\sec^2\dfrac{x}{2}$ c. $\sec^2\dfrac{x}{2}$ d. $-\sec^2\dfrac{x}{2}$

4. $\dfrac{1}{b}\tan^{-1}\left(\dfrac{x}{b}\right)+\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$

a. $-\dfrac{1}{x^2+b^2}+\dfrac{1}{x^2+a^2}$ b. $\dfrac{1}{x^2+b^2}+\dfrac{1}{x^2+a^2}$ c. $\dfrac{1}{x^2+b^2}-\dfrac{1}{x^2+a^2}$

d. $-\dfrac{1}{x^2+b^2}-\dfrac{1}{x^2+a^2}$

5. $\sec^{-1}x+\cos ec^{-1}\dfrac{x}{\sqrt{x^2-1}}$

a. $\dfrac{2}{\sqrt{x^2-1}}$ b. $\dfrac{-2}{\sqrt{x^2-1}}$ c. $\dfrac{1}{x\sqrt{x^2-1}}$ d. $\dfrac{2}{x\sqrt{x^2-1}}$

23 Let x = f(t) and y = g(t) be the parametric form with t as a parameter, then $\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}$ = $\dfrac{g^1(t)}{f^1(t)}$ where $f^1(t)\neq 0$

Based on the above information, answer the following questions

1.The derivative of f(tanx)w.r.t g(secx) at x=π/4, where $f^1(1)=2$ and $g^1(\sqrt{2})=4$ is

a.1/√2 b.√2 c.0 d.1

2. The derivative of $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ w.r.t $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ is

a.1 b.-1 c.2 d.4

3. The derivative of e^{x^3} w.r.t logx is

a. e^{x^2} b. $e^{x^3}.3x^2.2$ c. $e^{x^3}.3x^3$ d. $e^{x^2}.3x^2+3x$

4. The derivative of $\cos^{-1}(2x^2-1)$ w.r.t $\cos^{-1}x$ is

a.2 b. $\dfrac{-1}{2\sqrt{1-x^2}}$ c.2/x d.$1-x^2$

5.If y=$\dfrac{u^4}{4}$ *and* $u=\dfrac{2}{3}x^3$,then find dy/dx

a. $\dfrac{2}{27}x^9$ b. $\dfrac{16}{27}x^{11}$ c. $\dfrac{8}{27}x^9$ d. $\dfrac{2}{27}x^{11}$

24 Logarithmic differentiationis a powerful technique to differentiate functions of the form $f(x)=u(x)^{v(x)}$ where both u and v are differentiable functions of x.Let the function $y=f(x)=u(x)^{v(x)}$, then $y^1=u(x)^{v(x)}\left[v(x)\dfrac{u^1(x)}{u(x)}+\log u(x)v^1(x)\right]$

Based on the above information, answer the following questions

1. Differentiate x^x

    a.$x^x(1+\log x)$ b. $x^x(1-\log x)$ c. $-x^x(1+\log x)$ d. $x^x \log x$

2. Differentiate $x^x+a^x+x^a+a^a$

    a.$1+\log x+a^x \log a+ax^{a-1}$ b. $x^x(1+\log x)+\log a+ax^{a-1}$ c. $x^x(1+\log x)+x^a \log x+ax^{a-1}$

    d. . $x^x(1+\log x)+a^x \log a+ax^{a-1}$

3.If x= $e^{x/y}$, then find dy/dx

a. $-\frac{(x+y)}{x\log x}$ b. $-\frac{(x-y)}{x\log x}$ c. $\frac{(x+y)}{x\log x}$ d. $\frac{(x-y)}{x\log x}$

4.If $y=(2-x)^3(3+2x)^5$, then find dy/dx

a. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}-\frac{8}{2-x}\right]$ b. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}+\frac{3}{2-x}\right]$

c. $(2-x)^3(3+2x)^5\left[\frac{10}{3+2x}-\frac{3}{2-x}\right]$ d. $(2-x)^3(3+2x)^5\left[\frac{10}{3+2x}+\frac{3}{2-x}\right]$

5.If y= $x^x e^{2x+5}$, then find dy/dx

a. $x^x e^{2x+5}$ b. $x^x e^{2x+5}(3-\log x)$ c. $x^x e^{2x+5}(1-\log x)$

d. $x^x e^{2x+5}(3+\log x)$

25 Let f(x) be a real valued function, then its left hand derivative(L.H.D) at the point a is $f^1(a-)=\lim_{x\to 0}\frac{f(a-h)-f(a)}{-h}$ and its Right hand derivative(R.H.D) at the point a is $f^1(a+)=\lim_{x\to 0}\frac{f(a+h)-f(a)}{h}$,also a function f(x)is said to be differentiable at x=a and if its L.H.D and R.H.D at x=a exist and are equal. For the function

$$f(x)=\begin{cases} |x-3| & x\geq 1 \\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4} & x<1 \end{cases}$$

1. L.H.D of f(x)at x=1

    a.1 b.-1 c.0 d.2

2. f(x) is not differentiable at

    a.x=1 b.x=2 c.x=3 d.x=4

3.Find the value of $f^1(2)$

    a.1 b.2 c.3 d.4

4.Find the value of $f^1(-1)$

    a.x=1 b.x=2 c.x=-2 d.x=-1

5.R.H.D of f(x) at x=1 is

    a.1 b.-1 c.0 d.2

IV.LONG ANSWER QUESTIONS

26. If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$
27. If y= $e^{m\sin^{-1}x}$, then show that $(1-x^2)y_2-xy_1-m^2y=0$ (hint $(1-x^2)^{1/2}y_1=my$)
28. If x= sint, y=sinpt, prove that $(1-x^2)y_2-xy_1+p^2y=0$

    (hint $y_{1=}$pcospt/cost y_2=$\frac{-p^2 sinptcost+pcosptsint}{(cost)^3}$)
29. If y=log(x+$\sqrt{x^2+a^2}$), show that $(x^2+a^2)y_2+xy_1=0$
30. If x= a(cost+logtan$\frac{t}{2}$), y= asint evaluate $\frac{d^2y}{dx^2}$ at t=π/3

    (hint $\frac{dy}{dx}=tant, \frac{d^2y}{dx^2}=\frac{sint}{acos^4t}$)
31. If y = $x^{sinx-cosx}+\frac{x^2-1}{x^2+1}$, find $\frac{dy}{dx}$
32. If y= $(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$ (hint $(1+x^2)y_1=2\tan^{-1}x$)
33. Show that f(x)= |x-5| is continuous but not differentiable at x=5

    (for continuous LHL=RHL=f(5), for differentiability LHD=RHD)
34. If y= $(x+\sqrt{x^2+1})^m$, then show that $(x^2+1)y_2+xy_1-m^2y=0$
35. If x=acosθ+bsinθ, y = asinθ-bcosθ, then prove that $y^2y_2-xy_1+y=0$

    (Hint$\frac{dy}{dx}=-y/x$)

ANSWERS

MCQ

1.a 2.d 3.b 4.b 5.c

SHORT ANSWERS

6.discontinuous at x=2

8. a=3 b=2

10. k= -1

12. $\frac{-2(x+y)}{2x+3y^2}$

14. $e^{3x}\sin4x2^x(3+4\cot4x+\log2)$

15. $\frac{y(xlogy-y)}{x(ylogx-x)}$

16. $\frac{2^{x+1}log2}{1+4^x}$

17. $-\sin2y\cos^2y$

18. 2x

19. λ =n(n+1)

20. $\frac{e^{-\theta}(\theta^2-\theta^3+\theta+1)}{(\theta^2+\theta^3+\theta-1)}$

CASE STUDY

QN.NO	1	2	3	4	5
21	b	c	d	d	d
22	a	a	a	b	d
23	a	b	c	a	a

24	a	d	d	c	d
25	b	b	c	d	c

LONG ANSWERS

30. 8√3/a

31.$\frac{dy}{dx} = x^{sinx-cosx}$[(sinx-cosx)/x+logx(cosx+sinx)]+$\frac{4x}{(x^2+1)^2}$

APPLICATION OF DERIVATIVES

Key Points:

Derivative as Rate of Change

- Let y=f(x) be a function. Then $\frac{dy}{dx}$ denotes the rate of change of y w.r.t x.
- The value of $\frac{dy}{dx}$ at x = $\boldsymbol{x_0}$ i.e. $(\boldsymbol{dy/dx})_{x=x_0}$ represents the rate of change of y w.r.t x at x = $\boldsymbol{x_0}$
- If two variables x and y are varying with respect to another variable t, i.e., if x = f(t) and y= g(t), then by Chain Rule, $\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)$ provided $\frac{dx}{dt} \neq 0$
- $\frac{dy}{dx}$ is positive if y increases as x increases and is negative if y decreases as x increases.

Increasing and Decreasing Functions

- A function y = f (x) is said to be **increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in$ (a, b)

  Alternatively, a function y = f (x) is said to be increasing if $f'(x) \geq 0$ for each x in (a, b)

  (a) **strictly increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly increasing if $f'(x) > 0$ for each x in (a, b)

  (b) **decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ (a, b). Alternatively, a function y = f (x)) is said to be decreasing if $f'(x) \leq 0$ for each x in (a, b)

  (c) **strictly decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly decreasing if $f'(x)<0$ for each x in (a, b)

  (d) **constant** function in (a, b), if f (x) = c for all x ∈ (a, b), where c is a constant. Alternatively , f(x) is a constant function if $f'(x) = 0$.

- A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f.

Maxima And Minima

Definition: Let f be a function defined on an interval I. Then

1) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) > f(x)$, for all $x \in I$. The number f(c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.
2) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) < f(x)$, for all $x \in I$. The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.
3) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I. The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

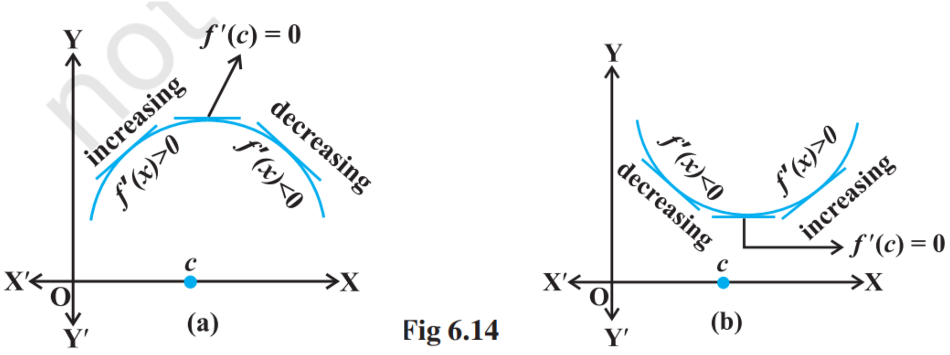
Local Maxima and Local Minima

Definition: Let f be a real valued function and let c be an interior point in the domain of f. Then

(a) c is called a point of local maxima if there is an $h > 0$ such that $f(c) \geq f(x)$, for all x in $(c - h, c + h)$, $x \neq c$. The value f(c) is called the local maximum value of f.

(b) c is called a point of local minima if there is an $h > 0$ such that $f(c) \leq f(x)$, for all x in $(c - h, c + h)$. The value f(c) is called the local minimum value of f.

Geometrically, the above definition states that if $x = c$ is a point of local maxima of f, then the graph of f around c will be as shown in Fig. below. Note that the function f is increasing (i.e., $f'(x) > 0$) in the interval $(c - h, c)$ and decreasing (i.e., $f'(x) < 0$) in the interval $(c, c + h)$. This suggests that $f'(c)$ must be zero,



(a) Fig 6.14 (b)

Theorem: Let f be a function defined on an open interval I. Suppose c ∈ I be any point. If f has a local maxima or a local minima at x = c, then either f ′(c) = 0 or f is not differentiable at c.

Definition: A point c in the domain of a function f at which either f ′(c) = 0 or f is not differentiable is called a critical point of f.

Theorem: (First Derivative Test) Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

1) If f ′(x) changes sign from positive to negative as x increases through c, i.e., if f ′(x) > 0 at every point sufficiently close to and to the left of c, and f ′(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.
2) If f ′(x) changes sign from negative to positive as x increases through c, i.e., if f ′(x) < 0 at every point sufficiently close to and to the left of c, and f ′(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.
3) If f ′(x) does not change sign as x increases through c, then c is neither a point of local maxima no a point of local minima. In fact, such a point is called point of inflection.

Theorem: (Second Derivative Test) Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then

1) x = c is a point of local maxima if f ′(c) = 0 and f ″(c) < 0 The value f (c) is local maximum value of f.
2) x = c is a point of local minima if f ′(c) 0 = and f ″(c) > 0 In this case, f (c) is local minimum value of f.
3) The test fails if f ′(c) = 0 and f ″(c) = 0. In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

Working rule for finding absolute maxima and/or absolute minima

Step 1: Find all critical points of f in the interval, i.e., find points x where either f ′(x) = 0 or f is not differentiable.
Step 2: Take the end points of the interval.
Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.
Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3. This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Multiple Choice Questions

1. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

A) 5/3 Sq.cm/sec
B) 7/3 Sq.cm/sec
C) 8/3 Sq.cm/sec
D) None of these

2. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate?

A) (1/3, 2/3)
B) (-1/3, 2/3)
C) (-1/3, -2/3)
D) (-1/2, -4/3)

3. The bottom of a rectangular swimming tank is 25 m by 40 m water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of water in the tank is rising.?

A) ¼ m/min
B) 2/3 m/min
C) 1/3 m/min
D) ½ m/min

4. The total revenue received from the sale of x units of a product is given by $R(x) = 10x^2 + 13x + 24$. Find the marginal revenue when x = 5

A) 113 Rupees
B) 223 Rupees
C) 93 Rupees
D) 339 Rupees

5. The interval in which the function $y = x^2e^{-x}$ is increasing is

A) $(-\infty, 0)$
B) (0,2)
C) $(2, \infty)$
D) None of these

Answers:

1) C 2) D 3) D 4) A 5) B

SHORT ANSWER QUESTIONS

6. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

7. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

8. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle?

9. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

10. Show that the function f given by f(x)=x^3-3x^2 + 7x, x € R, is strictly increasing on R.

11. Show that the function given by f(x) =e^{2x} is strictly increasing on R

12. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

13. The total cost C (x) in Rupees associated with the production of x units of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$$

Find the marginal cost when 17 units are produced.

14. Show that the function $f(x) = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

15. Prove that $y = \frac{4\,Sin\theta}{2+Cos\theta} - \theta \; is\ an\ increasing\ function\ of\ \theta\ in \left[0, \frac{\pi}{2}\right]$

16. Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.?

17.Find the maximum and the minimum values of the function $f(x) = x + 2, x \in (0,1)$?

18. Find the local maximum and the local minimum values of the function
$f(x) = \frac{-3}{4}\, x^4\ 8x^3 - \frac{45}{2}\, x^2 + 105$?

19. Find the maximum value of $f(x) = (x - 1)^{\frac{1}{3}}(x-2)$ in [1,9]

20. Find two numbers whose sum is 24 and whose product is as large as possible?

ANSWERS TO SHORT ANSWER QUESTIONS

6. Let x be the length of a side, V be the volume, and s be the surface area of the cube.Then, V = x^3 and S = $6x^2$ where x is a function of time t.

It is given that $\frac{dV}{dt} = 8 \text{ cm}^3/\text{s}$.

Then, by using the chain rule, we have:

$$8 = \frac{dV}{dt} = \frac{d}{dt}\left(x^3\right) = \frac{d}{dx}\left(x^3\right)\cdot\frac{dx}{dt} = 3x^2.\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{8}{3x^2} \qquad (1)$$

$$\text{Now, } \frac{dS}{dt} = \frac{d}{dt}\left(6x^2\right) = \frac{d}{dx}\left(6x^2\right)\cdot\frac{dx}{dt} \qquad \text{[By chain rule]}$$

$$= 12x.\frac{dx}{dt} = 12x\cdot\left(\frac{8}{3x^2}\right) = \frac{32}{x}$$

Thus, when x = 12 cm, $\frac{dS}{dt} = \frac{32}{12} \text{ cm}^2/\text{s} = \frac{8}{3} \text{ cm}^2/\text{s.}$

Hence, if the length of the edge of the cube is 12 cm, then the surface area is increasingat the rate of 8/3 cm^2/s.

Qn. 7) Let x be the length of a side and V be the volume of the cube. Then,V = $x^{3.}$

$\therefore \frac{dV}{dt} = 3x^2\cdot\frac{dx}{dt}$ (By chain rule)

It is given that,

$$\frac{dx}{dt} = 3 \text{ cm/s}$$

$$\frac{dV}{dt} = 3x^2(3) = 9x^2$$

∴

Thus, when x = 10 cm,

$$\frac{dV}{dt} = 9(10)^2 = 900 \text{ cm}^3/\text{s}$$

Hence, the volume of the cube is increasing at the rate of 900 cm^3/s when the edge is10 cm long.

Qn.8) Since the length (x) is decreasing at the rate of 5 cm/minute and the width (y) isincreasing at the rate of 4 cm/minute, we have:

$\frac{dx}{dt} = -5 \text{ cm/min}$ and $\frac{dy}{dt} = 4 \text{ cm/min}$

a) The perimeter (P) of a rectangle is given by,

(a) P = 2(x + y)

$$\therefore \frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-5+4) = -2 \text{ cm/min}$$

Hence, the perimeter is decreasing at the rate of 2cm/min

(b) The area (A) of a rectangle is given by,

A = x × y

$$\frac{dA}{dt} = \frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = -5y + 4x$$

When x = 8 cm and y = 6 cm,

$$\frac{dA}{dt} = (-5 \times 6 + 4 \times 8) \text{ cm}^2/\text{min} = 2 \text{ cm}^2/\text{min}$$

Hence, the area of the rectangle is increasing at the rate of 2 cm^2/min.

Qn9) Let y m be the height of the wall at which the ladder touches. Also, let the foot of the ladder be x m away from the wall. Then, by Pythagoras theorem, we have: $x^2 + y^2 = 25$[Length of the ladder = 5 m]

$$\Rightarrow y = \sqrt{25 - x^2}$$

Then, the rate of change of height (y) with respect to time (t) is given by,

$$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}} \cdot \frac{dx}{dt}$$

It is given that $\frac{dx}{dt} = 2 \text{ cm/s}$

.

$$\therefore \frac{dy}{dt} = \frac{-2x}{\sqrt{25-x^2}}$$

Now, when x = 4 m, we have:

$$\frac{dy}{dt} = \frac{-2\times 4}{\sqrt{25-4^2}} = -\frac{8}{3}$$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{8}{3}$ cm/s .

Qn. 10)

Sol.
$f(x)=x^2-3x^2+7x$
$f'(x) = 3x^2 - 6x +7$
in every interval of R.
$= 3(x^2 - 2x + 1) + 4$
$= 3(x-1)^2 +4> 0$,
.. the function fis strictly increasing on R. 1

Qn.11)

Sol.

$f(x) = e^{2x}$
$f'(x) = 2e^{2x}>0$, for all x = R.
:. f(x) is strictly increasing on R.

Qn. 12)

Soln

$$V = \frac{1}{3}\pi r^2 h$$

It is given that,

$$h = \frac{1}{6}r \Rightarrow r = 6h$$

$$\therefore V = \frac{1}{3}\pi\left(6h\right)^2 h = 12\pi h^3$$

The rate of change of volume with respect to time (t) is given by,

$$\frac{dV}{dt} = 12\pi \frac{d}{dh}\left(h^3\right)\cdot\frac{dh}{dt}$$

$$= 12\pi\left(3h^2\right)\frac{dh}{dt}$$

$$= 36\pi h^2\frac{dh}{dt}$$

$$\frac{dV}{dt} = 12 \text{ cm}^3/\text{s},$$

when h = 4 cm, we have:

$$12 = 36\pi(4)^2\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{12}{36\pi(16)} = \frac{1}{48\pi}$$

Hence, when the height of the sand cone is 4 cm, its height is increasing at the rateof $\frac{1}{48\pi}$ cm/s.

Qn. 13) Soln :-

Marginal cost (MC) $= \frac{dC}{dx} = 0.007\left(3x^2\right) - 0.003(2x) + 15$ When $= 0.021x^2 - 0.006x + 15$

∴

x = 17, MC = 0.021 (17^2) − 0.006 (17) + 15

= 0.021(289) − 0.006(17) + 15

= 6.069 − 0.102 + 15

= 20.967

Hence, when 17 units are produced, the marginal cost is ₹ 20.967.

14) Soln :-

$$f'(x) = \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{x^2}{(2+x)^2(1+x)} = \frac{(+)}{(+)(+)} = +ve$$

Since x>-1 , 1+x >0

Therefore the fn . f is st. increasing .

15) Soln:-

$$f'(\theta) = 4\left(\frac{(2+cos\theta)cos\theta - sin\theta(-sin\theta)}{(2+cos\theta)^2}\right) - 1$$

$$=4\left(\frac{2cos\theta + cos^2\theta + sin^2\theta}{(2+cos\theta)^2}\right) - 1$$

$$=4\frac{(2cos\theta+1)-(2+cos\theta)^2}{(2+cos\theta)^2}$$

$$=\frac{4cos\theta-cos^2\theta}{(2+cos\theta)^2}=\frac{cos\theta(4-cos\theta)}{(2+cos\theta)^2}=\frac{(+)(+)}{(+)}=+\text{ve}$$

Therefore ,the function f is an increasing function of x

16) Soln :-

Radius of sphere(r) = 2cm

V = 4/3 πr^3

$\frac{dV}{dr}$ = 4πr2

A = 4πr2

$\frac{dA}{dr}=8\pi r$

$$\frac{dV}{dA}=\frac{\frac{dV}{dr}}{\frac{dV}{dr}}$$

$$=\frac{4\pi r^2}{8\pi r}=\frac{r}{2}$$

$\frac{dV}{dA_{r=2}}=\frac{2}{2}$ =1 cm

17) $f(x) = x + 2$

$f\,'(x) = 1$

so for any value of x $f\,'(x)$ cannot be 0.

So $f(x)$ has no critical points.

Hence $f(x)$ has neither local maximum not local minimum

18) $f(x) = \frac{-3}{4}\ x^4\ 8x^3 - \frac{45}{2}\ x^2 + 105$

$f\,'(x) = -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

$= -3x(x + 3)(x + 5)$

$f''(x) = -9x^2 - 48x - 45$

$f\,'(x) = 0 \Longrightarrow -3x(x + 3)(x + 5) = 0$

$\Longrightarrow x = 0,\ x = -3,\ x = -5$

$f''(0)$=-45< 0

So x=0 is a local max. point.

f''(-3) =+18 >0

So x=-3 is a local min. point

f''(-5) =-30 < 0

So x=-5 is a local max. point.

19) $f(x) = (x\ -\ 1)^{\frac{1}{3}}(x - 2)$

$f\,'(x) = (x\ -\ 1)^{\frac{1}{3}}(1) + (x - 2) \times 1/\ 3\ (x - 1) -2/\ 3$

$= (x - 1) -2/\ 3\ (x - 1 + \frac{x-2}{3}) = \frac{x-1)^{\frac{-2}{3}}}{3}(3x - 3 + x - 2) =(\ 4x-5)/\ 3(x-1)\ 2/\ 3$

$f\,'(x) = 0 \Longrightarrow x = 5/\ 4$

Find the value of f at x= 5/4, x= 1 , &at x=9

$f(1) = 0$,
$f(9) = = 14$,
$f\ (5/\ 4\ ) < 0$
Maximum value of $f(x)$ is 14

20) Let x & y be the required nos.

X+y = 24 (given)

P = xy = x(24-x)

=24x -x^2

$\frac{dP}{dx}$=24-2x

$\frac{dP}{dx} = 0 \rightarrow$ 2x=24 or x=12

Second derivative of P is -2 which is -ve
So P is a max. when x=12

When x=12 ,y = 12
So the required numbers are x=12 & y= 12

LONG ANSWER QUESTIONS

21. Show that the volume of the greatest cylinder which can be inscribed in a cone of height 'h' and semi vertical angle α is $\frac{4}{27}\prod h^3 tan^2\alpha$

22. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone

23. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 sq units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units

24. Show that the right circular cylinder of given volume, open at the top, has minimum total surface area if its height is equal to the radius of the base

25. Find the volume of the largest right circular cylinder that can be inscribed in sphere of radius r centimeter

26. Show that the semi vertical angle of a right circular cone of maximum volume and given slant height is $tan^{-1}\sqrt{2}$

27. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area

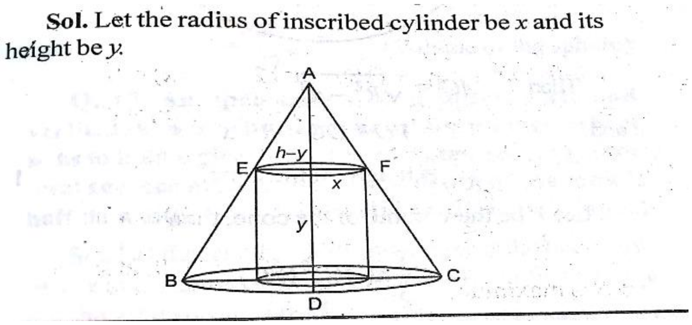
28. Show that the right circular cone of the least curved surface area and given volume has an altitude is equal to $\sqrt{2}$ times the radius of the base

29. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m. Find the dimensions of the rectangle that will produce the largest area of the window?

30. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere?

LONG ANSWER QUESTIONS – Solutions

21.



Sol. Let the radius of inscribed cylinder be x and its height be y.

V= $\pi x^2 y$

=$\pi(h-y)^2 \tan^2\alpha \cdot y$

$\frac{dv}{dx}$ = 0 $\rightarrow$ y =h or h=h/3

$\frac{d^2y}{dx^2} < 0$ at y =h/3

Then V=$\frac{4}{27}\prod h^3 tan^2\alpha$

Qn.22)

Let r and h be the radius and height of the cone .Let x and y be the radius and height of the cylinder.

Curved surface area of the cylinder S= $\frac{2\pi h}{r}$ (rx-x^2)

$\frac{ds}{dx}$ = 0 $\rightarrow$ x= r/2

$\frac{d^2s}{dx^2} < 0$ at x =r/3 , there S is maximum when x= r/2

Sol. Let x units be the side of the square base and y units be the height of the box.

$\therefore\ c^2 =$ Area of the base + Area of four walls

$$= x^2 + 4xy$$

$$\Rightarrow \quad y = \frac{c^2 - x^2}{4x} \qquad \text{...(i)} \quad \mathbf{1}$$

Now, volume of the box,

$$V = x^2 y$$

$$= \frac{x^2(c^2 - x^2)}{4x}$$

$$= \frac{1}{4}(c^2 x - x^3) \qquad \mathbf{1}$$

$$\therefore \quad \frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2)$$

$$\text{and} \quad \frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = -\frac{3}{2}x \qquad \mathbf{1}$$

Now, for maximum or minimum volume,

$$\frac{dV}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{4}(c^2 - 3x^2) = 0$$

$$\Rightarrow \quad x = \frac{c}{\sqrt{3}} \qquad \mathbf{1}$$

Also, at $x = \frac{c}{\sqrt{3}}$, $\frac{d^2V}{dx^2} = -\frac{3}{2} \cdot \frac{c}{\sqrt{3}} < 0$

$\therefore\ V$ is maximum at $x = \frac{c}{\sqrt{3}}$ **1**

and from (i), $y = \dfrac{c^2 - \frac{c^2}{3}}{4 \times \frac{c}{\sqrt{3}}} = \dfrac{c}{2\sqrt{3}}$

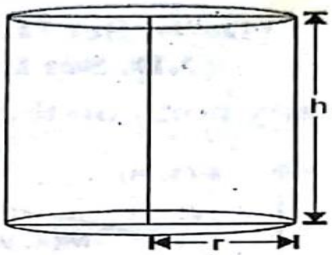
$\therefore$ Maximum value of $V = \frac{c^2}{3} \times \frac{c}{2\sqrt{3}}$

$$= \frac{c^3}{6\sqrt{3}} \qquad \mathbf{1}$$

Sol. Volume of a right circular cylinder,

$$V = \pi r^2 h$$

$$\Rightarrow \qquad h = \frac{V}{\pi r^2}$$



1

Total surface area of a right circular cylinder, open at the top,

$$S = \pi r^2 + 2\pi r h$$

$$\Rightarrow \qquad S = \pi r^2 + 2\pi r \frac{V}{\pi r^2}$$

$$\Rightarrow \qquad S = \pi r^2 + \frac{2V}{r} \qquad 1$$

For minimum S,

$$\frac{dS}{dr} = 0$$

and $$\frac{dS}{dr} = 2\pi r - \frac{2V}{r^2} \qquad 1$$

$$\Rightarrow \qquad 2\pi r - \frac{2V}{r^2} = 0$$

$$\Rightarrow \qquad r^3 = \frac{V}{\pi}$$

or $$r = \left(\frac{V}{\pi}\right)^{1/3} \qquad 1$$

Now, $$\frac{d^2S}{dr^2} = 2\pi + \frac{4V}{r^3}$$

At $r = \left(\frac{V}{\pi}\right)^{1/3}$,

$$\frac{d^2S}{dr^2} = 2\pi + 4\pi = 6\pi > 0$$

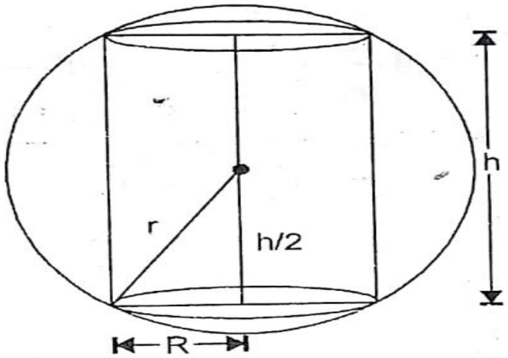
$\therefore$ At $r = \left(\frac{V}{\pi}\right)^{1/3}$, S is minimum 1

25.

Sol. Since a right circular cylinder of radius 'R' and height 'h' is inscribed in a sphere of radius 'r'. Therefore we have

$$r^2 = \left(\frac{h}{2}\right)^2 + R^2$$

$$\Rightarrow \qquad R^2 = \left(r^2 - \frac{h^2}{4}\right) \qquad \mathbf{1}$$



1

Volume of the cylinder,

$$V = \pi R^2 h$$

$$\Rightarrow \qquad V = \pi\left(r^2 - \frac{h^2}{4}\right)h$$

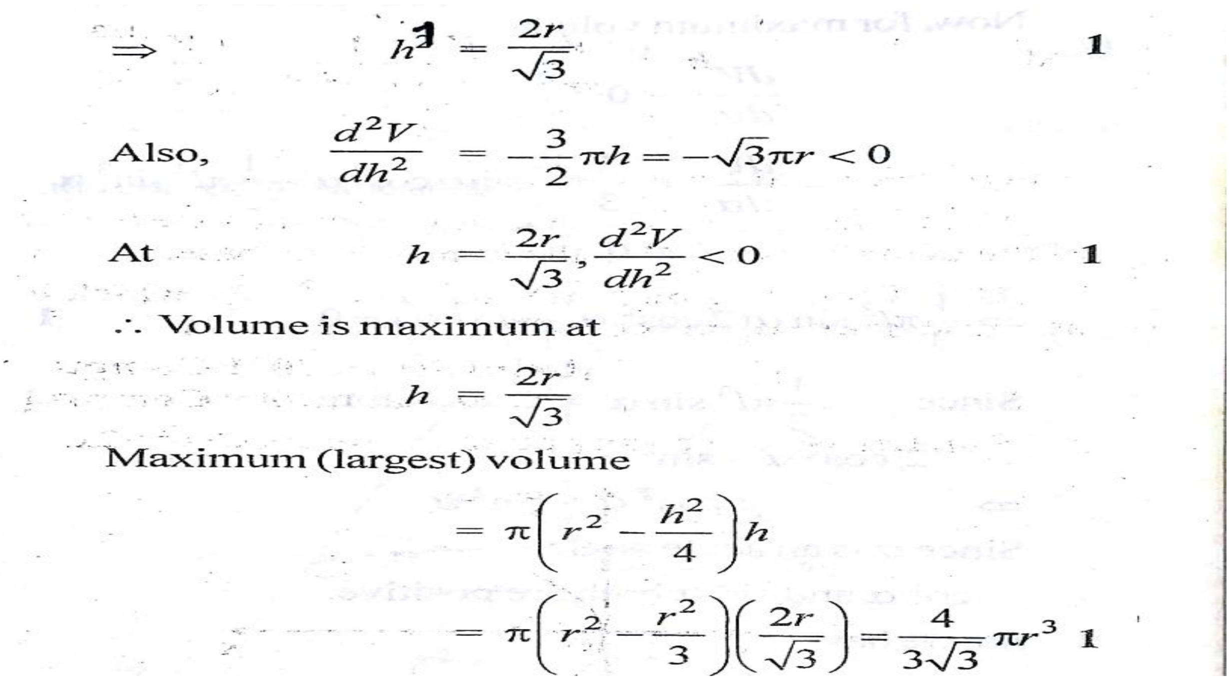
$$\Rightarrow \qquad V = \pi r^2 h - \frac{\pi}{4}h^3$$

For maximum V,

$$\frac{dV}{dh} = 0$$

$$\therefore \qquad \frac{dV}{dh} = \pi r^2 - \frac{3}{4}\pi h^2 = 0$$

$$\Rightarrow \qquad h^2 = \frac{4r^2}{3}$$



$$\Rightarrow \qquad h^{1} = \frac{2r}{\sqrt{3}}$$ 1

Also, $$\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h = -\sqrt{3}\pi r < 0$$

At $$h = \frac{2r}{\sqrt{3}}, \frac{d^2V}{dh^2} < 0$$ 1

∴ Volume is maximum at

$$h = \frac{2r}{\sqrt{3}}$$

Maximum (largest) volume

$$= \pi\left(r^2 - \frac{h^2}{4}\right)h$$

$$= \pi\left(r^2 - \frac{r^2}{3}\right)\left(\frac{2r}{\sqrt{3}}\right) = \frac{4}{3\sqrt{3}}\pi r^3$$ 1

26.

V=$\frac{1}{3}\pi r^2\theta$

l^2=$r^2 + h^2$

$r^{2=}l^2 - h^2$

So , V=1/3 π($l^2 - h^{2)}$ h

= $\frac{1}{3}$ π ($l^2h - h^3$

$\frac{dV}{dh}$ =$\frac{1}{3}$ π($l^2 - 3h^2$

$\frac{dV}{dh}$ =0 implies $l^2 = 3h^2$

Implies h = $\frac{l}{\sqrt{3}}$

$\frac{d2y}{dx2}$ = $\frac{1}{3}$ π(-6h) = -ve

So V is max. when h = $\frac{l}{\sqrt{3}}$

When V is max. , tan $\theta = \frac{r}{h} = \frac{l^2 - \frac{l^2}{3}}{\frac{l}{\sqrt{3}}} = \sqrt{2}$

So $\theta = \tan^{-1}\sqrt{2}$

Qn. 27) Let ABCD be a Rectangle inscribed in a given circle with centre at O and radius 'a'

Let AB= 2x and BC =2y

Then y= $\sqrt{a^2 - x^2}$

A= 4xy = 4x$\sqrt{a^2 - x^2}$

$\frac{dA}{dx} = 4\ (\frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}\ )$

$\frac{dA}{dx}$ =0 implies x=a/2 then y= a/2

$\frac{d2A}{dx2} < 0$ at x=y

Hence area is maximum when the rectangle is a square

28.Let r be the radius,h be the height and l be the slant height of the cone.

V = 1/3 πr^2h

h =$\frac{3V}{\pi r^2}$

curved surface area S= πrl

$S^2 = \pi^2\, r^2 l^2$

$= \pi^2\, r^2 (h^2\ + l^2)$

$Z = \pi^2\, r^2 (\frac{9v^2}{\pi^2\, r^4} + r^2)$

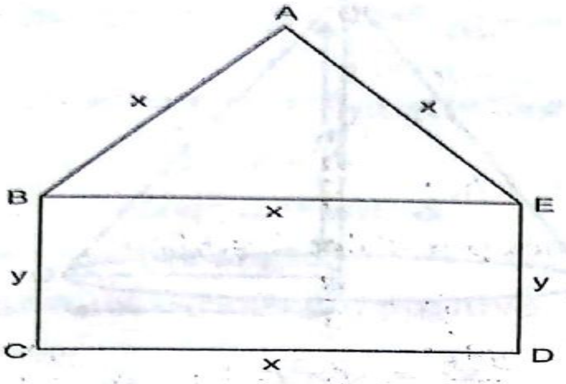
$\frac{dz}{dr}$ =0 implies $V^2 = \frac{4\pi^2\, r^6}{18}$

Then h=$\sqrt{2}$r when $\frac{d2z}{dr2} > 0$

29.

OR

Let the side of the equilateral triangle = x and the breadth of the rectangle = y.



Therefore the perimeter of the complete window

$$3x + 2y = 12 \quad \ldots(1)$$ 1

Its area, $$A = \frac{\sqrt{3}}{4}x^2 + xy$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}x^2 + \frac{x}{2}[12 - 3x]$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}x^2 + 6x - \frac{3}{2}x^2$$

$$\Rightarrow \quad A = (\sqrt{3} - 6)\frac{x^2}{4} + 6x$$ 1

Differentiating w.r.t. x, we get

$$\frac{dA}{dx} = \frac{x}{2}(\sqrt{3} - 6) + 6 \quad \ldots(2)$$ 1

For maxima or minima of A,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \quad \frac{x}{2}(\sqrt{3} - 6) + 6 = 0$$ 1

$$\Rightarrow \quad x = \left(\frac{12}{6 - \sqrt{3}}\right)\left(\frac{6 + \sqrt{3}}{6 + \sqrt{3}}\right)$$

$$x = \frac{12(6 + \sqrt{3})}{33} = \frac{4}{11}(6 + \sqrt{3})$$

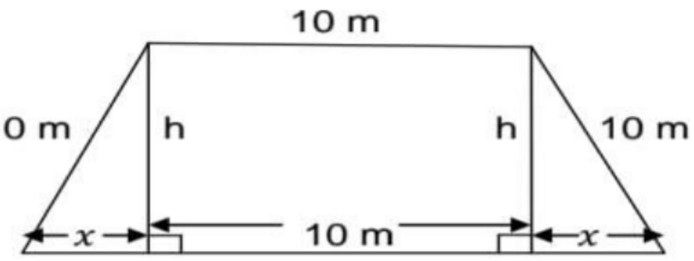
Differentiating equation (2) again w.r.t. x, we get

$$\frac{d^2A}{dx^2} = \frac{1}{2}(\sqrt{3} - 6) < 0$$ 1

Hence the area is maximum when y=$\frac{30-6\sqrt{3}}{11}$, x=$\frac{24+4\sqrt{3}}{11}$

CASE STUDY QUESTIONS

31) The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10m each. The height of the gate is h meter. On the basis of this information and figure given below answer the following questions:



(i) . The area A of the gate expressed as a function of x is

a. (10+x)$\sqrt{100+x^2}$

b. (10-x)$\sqrt{100+x^2}$

c. (10+x)$\sqrt{100-x^2}$

d. (10-x)$\sqrt{100-x^2}$

(ii) . The value of $\frac{dA}{dx}$ is

a. $\frac{2x^2+10x-100}{\sqrt{100-x^2}}$

b. $\frac{2x^2-10x-100}{\sqrt{100-x^2}}$

c. $\frac{2x^2+10x+100}{\sqrt{100-x^2}}$

d. $\frac{-2x^2-10x+100}{\sqrt{100-x^2}}$

(iii). For which positive value of x , $\frac{dA}{dx}$ =0

a. 10

b. 5

c. 20

d. 15

(iv). If at the value of x where $\frac{dA}{dx}$ =0 area of trapezium is maximum then what is maximum area of trapezium ?

a. 25$\sqrt{3}$ sqm

b. 100$\sqrt{3}$ sqm

c. 75$\sqrt{3}$ sqm

d. 50$\sqrt{3}$ sqm

(v). If the area of trapezium is maximum then the value of $\frac{d^2A}{dx^2}$ is

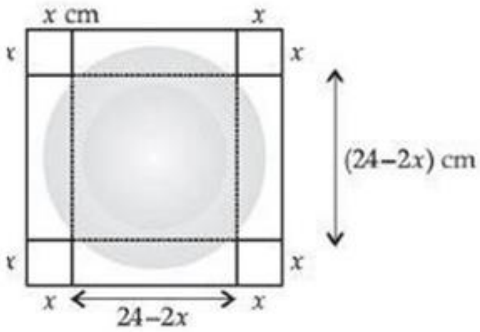
a. positive

b. negative

c. 0

d. none of these

32)An open box is to be made out of a piece of cardboard measuring 24 cm x 24 cm



by cutting of equal squares from the corners and turning up the sides.

Based on this information answer all the following Questions.

(i) The volume V(x) of the open box is

a) $4x^3 - 96x^2 + 576x$

b) $4x^3 + 96x^2 + 576x$

c) $2x^3 - 48x^2 + 288x$

d) $2x^3 + 48x^2 + 288x$

(ii) The value of dV/dx is

a) $12(x^2+16x-48)$
b) $12(x^2-16x+48)$
c) $12(x^2-16x-48)$
d) $12(x^2+16x+48)$

(iii) The value of d^2V/dx^2 is

a) 24(x+8)
b) 12(x-4)
c) 24(x-8)
d) 12(x+4)

(iv) For what value of the height, the volume of the open box is maximum

a) 3 cm

b) 9 cm

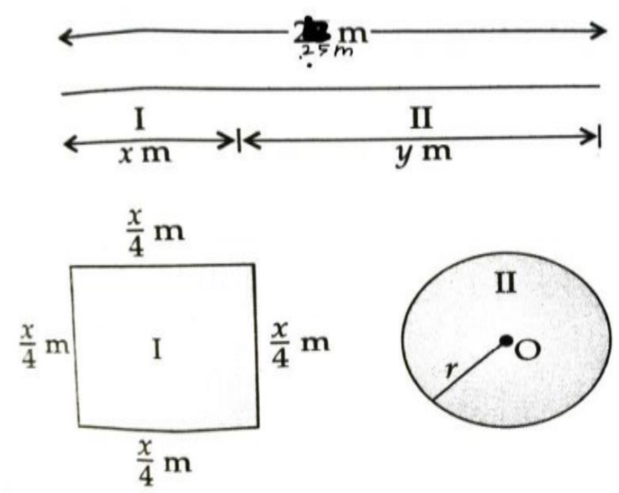
c) 1 cm

d) 4cm

(v) The volume is minimum if

a) $dV/dx=0$ and $d^2V/dx^2=0$
b) $dV/dx=0$ and $d^2V/dx^2<0$
c) $dV/dx=0$ and $d^2V/dx^2>0$
d) None of these

33) While constructing a house, a piece of wire of length 25cm is to be cut into pieces one of which is to bent into the form of a square and other into the form of a circle for the construction of two windows.



Based on the above information, answer the following question:

(i) What is the total area of the square and circle?

(a) $(x/4)^2 + \pi r^2$

(b) $(x/2)^2 + \pi r^2$

(c) $(x/4)^2 + \pi r$

(d) $(x/2)^2 + \pi r$

(ii) What is the relation of r with y?

(a) r = $y\ \pi$

(b) r = $y^2\pi$

(c) r = $xy\ \pi$

(d) r = $xy^2\pi$

(iii) If we talk about total length of wires then what is the relation between x and y?

(a) $x+y=25$

(b) $x+y=28$

(c) $x+y=26$

(d) $x+y=27$

(iv) When $dA\ dy = 0$, then find the value of y

(a) $50\pi\ \pi+4$ (ii) $75\ \pi+8$ (c) $25\pi\ \pi+4$ (d) $100\ \pi+8$

(v) Again, when $\frac{dA}{dy} = 0$, then the value of x

. (a) $50\ \pi+4$

(b) $100\ \pi+4$

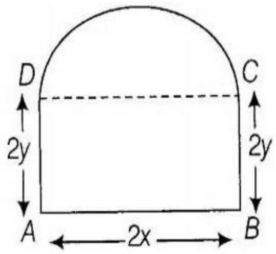
(c) $25\ \pi+4$

(d)$50\pi\ \pi+4$

5.The sum of the length hypotenuse and a side of a right-angled triangle is given by AC+BC = 10

34)

An architect designs a building for a small company . The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi circular opening. The window is having a perimeter of 10 meter as shown in the figure.



Based on the above information answer the following:

(i)	**If 2x and 2y represents the length and breadth of the rectangular portion of the window, then the relation between the variables is** **(a) 4y – 2x = 10- π (b) 4y = 10- (2-π)x** **(c) 4y = 10- (2+π)x (d) 4y – 2x = 10 + π**	1
(ii)	**The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is** **(a) A= 10x + $(2 + \frac{\pi}{2})\ x^2$ (b) A= 10x - $(2+\frac{\pi}{2})\ x^2$** **(c) A= 10x - $(2-\frac{\pi}{2})\ x^2$ (d) A = 4xy + $\frac{\pi}{2}x^2$, where y= $\frac{5}{2} + \frac{1}{2}(2+\pi)x$**	1
(iii)	**The maximum value of area of the whole window, A is** **(a) A = $\frac{50}{4+\pi}cm^2$ (b) A = $\frac{50}{4+\pi}m^2$ (c) A = $\frac{100}{4+\pi}m^2$ (d) A = $\frac{50}{4-\pi}\ m^2$**	1
(iv)	**The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, the length of the rectangular portion of the window should be** **(a) $\frac{20}{4+\pi}$ m (b) $\frac{10}{4+\pi}$ m (c) $\frac{4}{10+\pi}$ m (d) $\frac{100}{4+\pi}$ m**	1

35) Scientist want to know the Oil- Reserves in sea so they travel over the sea along the curve f(x) = $(x+1)^3\ (x-3)^3$ by an airplane. A student of class XII discuss the characteristic of the curve. Answer the following questions on the basis of the information given above

(i) The first order derivative of the given function is

(a) $3(x+1)^2(x-3)^2$

(b) $6(x+1)^2(x-3)^2(x-1)$

(c) 2(x-1)

(d) None of these

(ii) The critical point of the given function are

(a) -1,1,3

(b) 1,3,-2

(c) 1,2

(d) None of these

(iii) The interval in which the given function is strictly increasing is

(a) (1,3) U (3,∞)

(b) (-∞, −1)U(-1,1)

(c) (1,3) U (-1,∞)

(d) None of these

(iv) The interval in which the given function is decreasing is

(a) (1,3) U (3,∞)

(b) (-∞, −1)U(-1,1)

(c) (1,3) U (-1,∞)

(d) None of these

ANSWERS TO CASE STUDY QUESTIONS

Qn.31 (i) c (ii) d (iii) b (iv) c (v) b

Qn.32 (i) a (ii) b (iii) c (iv) d (v) c

Qn.33 (i) a (ii)b (iii) a (iv) c (v) b

Qn34 (i) (ii) (iii) (iv) (v)

Qn.35 (i) b (ii) a (iii)a (iv) b

CHAPTER 7 – INTEGRALS

DEFINITION OF INTEGRALS

Let $\frac{d}{dx}F(x) = f(x)$. Then we write $\int f(x)\,dx = F(x) + C$. These integrals are called indefinite integrals or general integrals, C is called constant of integration. All these integrals differ by a constant.

PROPERTIES OF INTEGRALS

(I) The process of differentiation and integration are inverses of each other in the sense of the following results :

$$\frac{d}{dx}\int f(x)\,dx = f(x)$$

and $\int f'(x)\,dx = f(x) + C$, where C is any arbitrary constant.

(II) Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.

(III) $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(IV) For any real number k, $\int k\, f(x)\,dx = k\int f(x)\,dx$

(V) Properties (III) and (IV) can be generalised to a finite number of functions $f_1, f_2, \ldots, f_n$ and the real numbers, $k_1, k_2, \ldots, k_n$ giving

$$\int [k_1 f_1(x) + k_2 f_2(x) + \ldots + k_n f_n(x)]\,dx$$

$$= k_1\int f_1(x)\,dx + k_2\int f_2(x)\,dx + \ldots + k_n\int f_n(x)\,dx.$$

PRE-REQUISITE FORMULAS :

1. $sin^3x \;=\; \frac{3\,sinx - sin3x}{4}$

2. $cos^3x \;=\; \frac{3\,cosx + cos3x}{4}$

3.cos^2x $= \frac{1+\cos 2x}{2}$

4. sin^2x $= \frac{1-\cos 2x}{2}$

5. 2 cosx cosy= cos (x+y) +cos (x-y)

6. 2 sinx sin y = cos (x-y) – cos (x+y)

7. 2sinx cosy= sin(x+y) + sin (x-y)

8. 2cosx siny= sin (x+y) – sin (x-y)

3 .STANDARD INTEGRALS

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$. Particularly, $\int dx = x + C$

(ii) $\int \cos x \, dx = \sin x + C$

(iii) $\int \sin x \, dx = -\cos x + C$

(iv) $\int \sec^2 x \, dx = \tan x + C$

(v) $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

(vi) $\int \sec x \tan x \, dx = \sec x + C$

(vii) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$

(viii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(ix) $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

(x) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

(xi) $\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$

(xii) $\int e^x dx = e^x + C$

(xiii) $\int a^x dx = \frac{a^x}{\log a} + C$

(xiv) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

(xv) $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$

(xvi) $\int \frac{1}{x} dx = \log|x| + C$

4.PARTIAL FRACTIONS

1. $\dfrac{px+q}{(x-a)(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{x-b}, \ a \neq b$

2. $\dfrac{px+q}{(x-a)^2} = \dfrac{A}{x-a} + \dfrac{B}{(x-a)^2}$

3. $\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \dfrac{A}{x-a} + \dfrac{B}{x-b} + \dfrac{C}{x-c}$

4. $\dfrac{px^2+qx+r}{(x-a)^2(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{(x-a)^2} + \dfrac{C}{x-b}$

5. $\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \dfrac{A}{x-a} + \dfrac{Bx+C}{x^2+bx+c}$

where $x^2 + bx + c$ can not be factorised further.

5. INTEGRATION BY SUBSTITUTION

(i) $\int \tan x \, dx = \log|\sec x| + C$

(ii) $\int \cot x \, dx = \log|\sin x| + C$

(iii) $\int \sec x \, dx = \log|\sec x + \tan x| + C$

(iv) $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$

6. INTEGRATION OF SPECIAL FUNCTIONS

(i) $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + C$

(ii) $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + C$

(iii) $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + C$

7. INTEGRATION BY PARTS

For given functions f_1 and f_2, we have

$$\int f_1(x) \cdot f_2(x)\,dx = f_1(x)\int f_2(x)\,dx - \int\left[\frac{d}{dx}f_1(x)\cdot\int f_2(x)\,dx\right]dx$$

$$\int e^x[f(x) + f'(x)]\,dx = \int e^x f(x)\,dx + C$$

8. INTEGRATION OF SPECIAL FUNCTIONS

(i) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left| x + \sqrt{x^2 - a^2} \right| + C$

(ii) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left| x + \sqrt{x^2 + a^2} \right| + C$

(iii) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

(iv) Integrals of the types $\int \frac{dx}{ax^2 + bx + c}$ or $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$ can be transformed into standard form by expressing

$$ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right]$$

(v) Integrals of the types $\int \frac{px + q\, dx}{ax^2 + bx + c}$ or $\int \frac{px + q\, dx}{\sqrt{ax^2 + bx + c}}$ can be

transformed into standard form by expressing

$px + q = A\frac{d}{dx}(ax^2 + bx + c) + B = A\ (2ax + b) + B$, where A and B are determined by comparing coefficients on both sides.

DEFINITE INTEGRALS

9.PROPERTIES OF DEFINITE INTEGRALS

P₀ : $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

P₁ : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$. In particular, $\int_a^a f(x)\,dx = 0$

P₂ : $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

P₃ : $\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$

P₄ : $\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$
(Note that P_4 is a particular case of P_3)

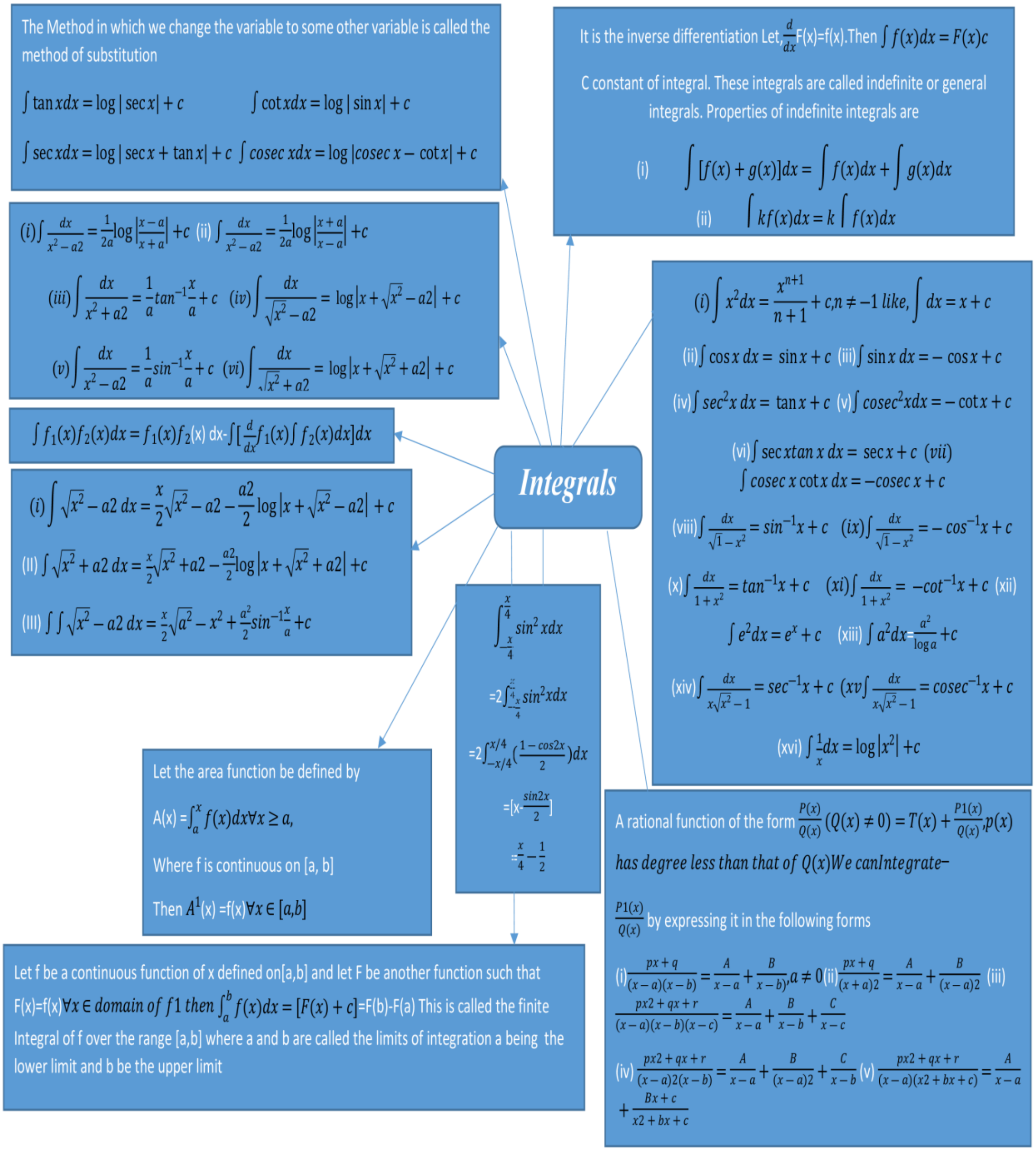
P₅ : $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a - x)\,dx$

P₆ : $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(2a - x) = f(x)$ and
0 if $f(2a - x) = -f(x)$

P₇ : (i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)\,dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

MIND MAPPING :



Integrals

The Method in which we change the variable to some other variable is called the method of substitution

$\int \tan x dx = \log|\sec x| + c$ $\int \cot x dx = \log|\sin x| + c$

$\int \sec x dx = \log|\sec x + \tan x| + c$ $\int cosec\, x dx = \log|cosec\, x - \cot x| + c$

(i)$\int \frac{dx}{x^2-a2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$ (ii) $\int \frac{dx}{x^2-a2} = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c$

$(iii)\int \frac{dx}{x^2+a2} = \frac{1}{a}tan^{-1}\frac{x}{a} + c$ $(iv)\int \frac{dx}{\sqrt{x^2-a2}} = \log\left|x + \sqrt{x^2-a2}\right| + c$

$(v)\int \frac{dx}{x^2-a2} = \frac{1}{a}sin^{-1}\frac{x}{a} + c$ $(vi)\int \frac{dx}{\sqrt{x^2+a2}} = \log\left|x + \sqrt{x^2+a2}\right| + c$

$\int f_1(x)f_2(x)dx = f_1(x)f_2(x)$ dx-$\int[\frac{d}{dx}f_1(x)\int f_2(x)dx]dx$

$(i)\int \sqrt{x^2-a2}\, dx = \frac{x}{2}\sqrt{x^2-a2} - \frac{a2}{2}\log\left|x + \sqrt{x^2-a2}\right| + c$

(II) $\int \sqrt{x^2+a2}\, dx = \frac{x}{2}\sqrt{x^2}+a2 - \frac{a2}{2}\log\left|x + \sqrt{x^2}+a2\right| + c$

(III) $\int\int \sqrt{x^2-a2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a} + c$

Let the area function be defined by

A(x) =$\int_a^x f(x)dx \forall x \geq a,$

Where f is continuous on [a, b]

Then A^1(x) =f(x)$\forall x \in [a,b]$

$\int_{-\frac{x}{4}}^{\frac{x}{4}} sin^2 xdx$

$=2\int_{-\frac{x}{4}}^{\frac{x}{4}} sin^2xdx$

$=2\int_{-x/4}^{x/4}(\frac{1-cos2x}{2})dx$

=[x-$\frac{sin2x}{2}$]

$=\frac{x}{4}-\frac{1}{2}$

Let f be a continuous function of x defined on[a,b] and let F be another function such that F(x)=f(x)$\forall x \in domain\ of\ f1\ then \int_a^b f(x)dx = [F(x)+c]$=F(b)-F(a) This is called the finite Integral of f over the range [a,b] where a and b are called the limits of integration a being the lower limit and b be the upper limit

It is the inverse differentiation Let,$\frac{d}{dx}$F(x)=f(x).Then $\int f(x)dx = F(x)c$

C constant of integral. These integrals are called indefinite or general integrals. Properties of indefinite integrals are

(i) $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$

(ii) $\int kf(x)dx = k\int f(x)dx$

$(i)\int x^2dx = \frac{x^{n+1}}{n+1} + c, n \neq -1\ like, \int dx = x + c$

(ii)$\int \cos x\, dx = \sin x + c$ (iii)$\int \sin x\, dx = -\cos x + c$

(iv)$\int sec^2x\, dx = \tan x + c$ (v)$\int cosec^2xdx = -\cot x + c$

(vi)$\int \sec x \tan x\, dx = \sec x + c$ (vii) $\int cosec\, x \cot x\, dx = -cosec\, x + c$

(viii)$\int \frac{dx}{\sqrt{1-x^2}} = sin^{-1}x + c$ $(ix)\int \frac{dx}{\sqrt{1-x^2}} = -cos^{-1}x + c$

(x)$\int \frac{dx}{1+x^2} = tan^{-1}x + c$ $(xi)\int \frac{dx}{1+x^2} = -cot^{-1}x + c$ (xii) $\int e^2dx = e^x + c$ (xiii) $\int a^2dx = \frac{a^2}{\log a} + c$

(xiv)$\int \frac{dx}{x\sqrt{x^2-1}} = sec^{-1}x + c$ $(xv\int \frac{dx}{x\sqrt{x^2-1}} = cosec^{-1}x + c$

(xvi) $\int \frac{1}{x}dx = \log\left|x^2\right| + c$

A rational function of the form $\frac{P(x)}{Q(x)}(Q(x) \neq 0) = T(x) + \frac{P1(x)}{Q(x)}, p(x)$ $has\ degree\ less\ than\ that\ of\ Q(x) We\ can Integrate-$

$\frac{P1(x)}{Q(x)}$ by expressing it in the following forms

(i)$\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}, a \neq 0(ii)\frac{px+q}{(x+a)2} = \frac{A}{x-a} + \frac{B}{(x-a)2}$ (iii) $\frac{px2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

(iv) $\frac{px2+qx+r}{(x-a)2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)2} + \frac{C}{x-b}$ (v) $\frac{px2+qx+r}{(x-a)(x2+bx+c)} = \frac{A}{x-a} + \frac{Bx+c}{x2+bx+c}$

MULTIPLE CHOICE QUESTIONS:

1. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

a.2π b. $\frac{\pi}{12}$ c. 4 d. $\frac{\pi}{3}$

2. $\int_0^{\frac{\pi}{2}} log\frac{4+3sinx}{4+3cosx}$ dx equals

a.2 b. ¾ *c. 0* d. -2

3. $\int_e^{e^2}\frac{dx}{xlogx}$ equal to

a.2 b. *log2* *c. 0* d. 1

4. $\int_{-1}^{1}\left|x\right|dx$ is

2 b. -1 c. 1 d. 0

5. $\int_0^{\frac{\pi}{4}}\frac{cosx}{sin^2x}$dx is

a.2 b. $\sqrt{2}$ c. 1 d. -$\sqrt{2}$

SHORT ANSWER QUESTIONS :

1) Find $\int\frac{\log(sinx)}{tanx}$ dx
2) Find $\int\frac{\sqrt{tanx}}{\sin x\cos x}$ dx
3) $\int tan^8xsec^4$dx
4) $\int x^2\ logx\ dx$
5) $\int tan^4x\ \ dx$= 1/3tan^3x-tanx +x +C
6) Evaluate $\int\frac{Cos2x-Cos2\propto}{Cosx-Cos\propto}\ \ dx$
7) Evaluate $\int_2^8|x-5|$dx
8) Find $\int\sqrt{10-4x+4x^2}$ dx
9) Find$\int x\ sin^{-1}x\ dx$
10) Find $\int_0^{\frac{\pi}{4}}\frac{1+tanx}{1-tanx}\ dx$
11) Evaluate $\int e^x\left(\frac{1+sinx}{1+cosx}\right)dx$
12) Find $\int\frac{x^3}{x^4+3x^2+2}$ dx
13) Evaluate $\int\frac{dx}{2sin^2x+5\ cos^2x}$
14) $\int\frac{e^x}{(1+e^x)(2+e^x)}$ dx
15) $\int\frac{(x^2+1)e^x}{(x+1)^2}\ dx$
16) $\int\frac{1}{\cos(x-a)\cos(x-b)}dx$
17) $\int\frac{e^x}{e^{2x}+6e^x+8}$ dx
18) $\int\frac{1}{sinxcos^3x}\ dx$

19) $\int \frac{\sin(x-a)}{\sin(x+a)}$ dx

20) $\int \frac{dx}{x(x^4-1)}$

LONG ANSWER QUESTIONS:

1.$\int \frac{sin^8x-cos^8x}{1-2sin^2xcos^2x}$ dx

2. Evaluate$\int \frac{x+2}{2x^2+6x+5}\ dx.$

3.Evaluate$\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx

4. Find $\int \frac{x^2}{x^4+x^2-2}\ dx$

5. $\int \frac{2}{(1-x)(1+x^2)}dx$

6. $\int \frac{3x-1}{(x+2)^2}$ dx

7. Find $\int tan^{-1}\sqrt{\frac{1-x}{1+x}}$ dx

8. Evaluate $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}$ dx

9.Evaluate $\int_{-1}^2 |x^3-x|\ dx.$

10.$\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}$ dx

11. $\int_0^{\pi} \frac{x\ tanx}{secx+tanx}$dx

12.$\int_0^{\pi/2} \frac{sin^2x}{sinx+cosx}$ dx

13.$\int \frac{x^4}{(x+1)(x^2+1)}$dx

14. $\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx

15. $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+ cos^4x}$ dx

CASE STUDY QUESTIONS :

CASE STUDY 1:

For any function f(x) , we have

$$\int_a^b f(x)\ dx = \int_a^{c_1} f(x)\ dx + \int_{c_1}^{c_2} f(x)\ dx + \cdots + \int_{c_n}^b f(x)\ dx \quad ; \text{where } a < c_1 < c_2 < \cdots . < c_n < b$$

On the basis of the above information , answer the following questions

$i)\ \int_{-1}^{1} |x|\ dx$

(A)1 (B)2 (C) − 1 (D)0

$ii)\ \int_{0}^{2} |x-1|\ dx$

(A)2 (B)1 (C) − 1 (D)0

$iii)\ \int_{0}^{\pi} |\cos x|\ dx$

(A) $\frac{\pi}{2}$ (B)2 (C) $\frac{\pi}{4}$ (D)0

$iv)\ \int_{0}^{2} [x]\ dx$; where $[x]$ is the greatest integer function

(A)2 (B)1 (C) − 1 (D)0

CASE STUDY 2:

The given integral $\int f(x)dx$ can be transferred into another form by changing the independent variable 'x ' to 't' by substituting $x = g(t)$

Consider $I = \int f(x)dx$

Put $x = g(t)\ and\ \frac{dx}{dt} = g'(t)dt$

Thus $I = \int f(x)dx \ = \int f\big(g(t)\big)g'^{(t)}dt$

This change of variable formula is known as Integration by substitution.

$i)\ \int \frac{1}{\mathrm{x + x \log x}}\mathrm{dx}$

$\mathrm{(A) \log |1 + logx| + C}$ $\mathrm{(B) \log x + C}$ $\mathrm{(C) \log |x + xlogx| + C}$ $\mathrm{(D) \log |x + logx|}$

$$ii)\ \int \frac{(\sin^{-1} \mathrm{x})^2}{\sqrt{1-\mathrm{x}^2}}\,\mathrm{dx}$$

(A) $\frac{(\sin^{-1} \mathbf{x})^{\mathbf{2}}}{2} + \mathrm{C}$ (B) $\frac{(\sin^{-1} \mathrm{x})^3}{3} +$ (C) $\frac{\sin^{-1} \mathrm{x}}{2} + \mathrm{C}$ (D) $\sin^{-1} \mathrm{x} + \mathrm{C}$

$iii)\ \int \frac{\sin \sqrt{\mathbf{x}}}{\sqrt{\mathbf{x}}}\,\mathrm{dx}$

(A) $-2 \sin \sqrt{\mathrm{x}} + \mathrm{C}$ (B) $2 \cos \sqrt{\mathrm{x}} + \mathrm{C}$ (C) $-2 \cos \sqrt{\mathrm{x}} + \mathrm{C}$ (D) $\sqrt{\mathrm{x}} + \mathrm{C}$

$$iv)\ \int_0^1 \frac{e^{\sqrt{\mathbf{x}}}}{\sqrt{\mathbf{x}}}\,dx$$

(A) e (B) $2(\mathrm{e}-1)$ (C) $\mathrm{e}-1$ (D) $\mathrm{e}+1$

CASE STUDY 3:

The given integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting x = g(t)

Consider $I = \int f(x)dx$

Put x = g(t) so that $\frac{dx}{dt}$ = g'(t)

d(x) = g'(t) dt

Thus $I = \int f(x)dx = \int f\big(g(t)\big)g'(t)dt$

This change of variable formula is one of the important tools available in the name of integration by substitution

(i) $\int 2x \sin(x^2 + 1)dx$ is equal to

A) $\cos(x^2 + 1) + C$ B) $-\cos(x^2 + 1) + C$ C) $\sin(x^2 + 1) + C$

D) $-\sin(x^2 + 1) + C$

(ii) $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ is equal to

A) $-sin(\tan^{-1} x) + C$ B) $-cos(\tan^{-1} x) + C$ C) $cos(\tan^{-1} x) + C$ D) $sin(\tan^{-1} x) + C$

(iii) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

A) $\frac{(\sin^{-1} x)^2}{2}$ + C B) $\frac{(\cos^{-1} x)^2}{2}$ + C C) $\frac{(\tan^{-1} x)^2}{2}$ + C D)None of these

(iv) $\int \frac{sinx}{(1+cosx)^2}$ dx is equal to

A) sinx + C B) $\frac{1}{1+cosx}$ + C C) (1 + cosx) +C D) None of these

CASE STUDY 4:

"The integration of the product of two functions = (first function) x (integral of the second function) - Integral of [(differential coefficient of the first function) x (Integral of the second function)]". This is found quite useful in integrating product of functions .

Based on the above information answer the following.

(i) $\int x tan^{-1} x \, dx$ is equal to:

A) $\frac{x^2}{2}\tan^{-1} x + \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$ B) $\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} - \frac{\tan^{-1} x}{2} + C$

C) $\frac{x^2}{2}\tan^{-1} x + \frac{\tan^{-1} x}{2} + C$, D) $\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$

(ii) $\int logx \, dx$ is equal to:

A) x log x - x + C , B) x log x + x + C

C) log x - x + C, D) None of these

(iii) $\int x \, sec^2 x \, dx$ is equal to:

A) x tan x + log |sec x| + C B) x tan x - log |sec x| + C

C) tan x - log |sec x| + C D) None of these

(iv) $\int x \, log 2x \, dx$ is equal to:

A) $\frac{x^2}{2}$ log 2x $-\frac{x^2}{4}$ + C B) $\frac{x^2}{2}$ log 2x $+\frac{x^2}{4}$ + C

C) log 2x $-\frac{x^2}{4}$ + C D)None of these

CASE STUDY 5:

$$\int e^x(f'(x)+\ f(x))dx =\ \int e^x\ f'(x)dx + \ \int\ f(x)e^x dx$$

$$= \int e^x\ f'(x)dx +\ \ f(x)e^x - \ \int\ f'(x)e^x dx + \text{C}$$

$$=\ f(x)e^x + \text{C}$$

Based on the above information answer the following.

(i) $\int e^x(sinx+\cos x)\ dx$ is equal to:

A)$-e^x sinx+C$, B) $e^x cosx+C$, C)$e^x sinx+C$, D)None of these

(ii) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$ is equal to:

A) $\frac{e^x}{x}+C$, B) $\frac{e^{2x}}{x}+C$, C) $\frac{e^x}{2x}+C$, D) None of these

(iii)$\int e^x\left(\frac{1-sinx}{1-cosx}\right)dx$ is equal to:

A) $e^x cot\frac{x}{2}+C$, B) $e^{2x} cot\frac{x}{2}+C$, C)$-e^x cot\frac{x}{2}+C$, D)None of these

(iv)$\int e^x\left(\frac{x^2+1}{(x+1)^2}\right)dx$ is equal to:

A) $\frac{x-1}{x+1}e^x+C$, B) $\frac{x+1}{x-1}e^x+C$, C) $2x\ e^x+C$, D)None of these

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ANSWERS:

ANSWERS OF MCQ:

1.b 2. c 3. b 4.c 5. d

ANSWERS OF SHORT ANSWER QUESTIONS:

1.$\frac{(\log sinx)^2}{2}\ +c$ (Hint : put log (sinx) = t

2.$\int\frac{\sqrt{\tan x}}{\sin x\ cosx}\ dx = \int\frac{\sqrt{tanx}}{tanx}.sec^2\ x\ dx$

Let tanx = t, $\sec^2$x dx = dt

$\int \frac{1}{\sqrt{t}}$dt = $2\sqrt{t}$ = $2\sqrt{tanx}$+C

3. $\int tan^8 x sec^4\, dx = \int tan^8 x(tan^2$x +1) sec^2x dx =

$$= \int tan^{10} x sec^2 x\, dx \; + \int tan^8 x sec^2\, x\, dx \quad (\tan x = t, sec^2\, x\, dx = dt\,)$$

$$= \frac{tan^{11}\, x}{11} + \frac{tan^9 x}{9} + \text{C}$$

4. $\int x^2\, logx\, dx = \int logx . x^2\, dx = logx.\frac{x^3}{3} - \int \frac{1}{x}.\, \frac{x^3}{3}\, dx =$logx.$\frac{x^3}{3} - \frac{1}{3}\int x^2$dx

$$= \text{logx.}\frac{x^3}{3} - \frac{x^3}{9} + c$$

5.$\int tan^4 x\;\, dx$= 1/3tan^3x-tanx +x +C

6 $\int \frac{Cos2x - Cos2\propto}{Cosx - Cos\propto}\quad dx$ =2(sinx +x cosα)+C

(Hint : cos2x = $2cos^2 x - 1$ & cos2α = $2cos^2\alpha - 1$

7. $\int_2^8 |x-5|$dx = 9, (Hint : $\int_{-2}^{5} -(x-5)dx + \int_5^8 (x-5)dx$

$$\text{I} = \int \sqrt{10 - 4x + 4x^2}\, dx \quad = \int \sqrt{(2x-1)^2 + (3)^2}\;\, dx$$

Put $t = 2x - 1$, then $dt = 2dx$.

Therefore, $\text{I} = \frac{1}{2}\int \sqrt{t^2 + (3)^2}\; dt$

$$= \frac{1}{2}\, t\, \frac{\sqrt{t^2+9}}{2} + \frac{9}{4}\log\left|t + \sqrt{t^2+9}\right| + \text{C}$$

$$= \frac{1}{4}(2x-1)\sqrt{(2x-1)^2 + 9} + \frac{9}{4}\log\left|(2x-1) + \sqrt{(2x-1)^{\,2} + 9}\right| + \text{C}\,.$$

8.

9.

$$\int x\sin^{-1}x\,dx = \sin^{-1}x\cdot\frac{x^2}{2}-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cdot\sin^{-1}x}{2}+\frac{1}{2}\left\{\int\sqrt{1-x^2}\,dx-\int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$=\frac{x^2\cdot\sin^{-1}x}{2}+\frac{1}{2}\left\{\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right\}$$

or $\dfrac{x^2\cdot\sin^{-1}x}{2}+\dfrac{x\sqrt{1-x^2}}{4}-\dfrac{1}{4}\sin^{-1}x\ +C$

10. $\int_0^{-\frac{\pi}{4}}\frac{1+tanx}{1-tanx}\,dx=\int_0^{-\frac{\pi}{4}}tan\left(\frac{\pi}{4}+x\right)dx=\log\ sec\left(\frac{\pi}{4}+x\right)_0^{-\frac{\pi}{4}}$

$$= logsec0 - logsec\,\pi/4 \ = log1 - log\sqrt{2}$$

=0 - -$\frac{1}{2}$ $log2$ = -1/2 log 2

11. $\int e^x\left(\frac{1+sinx}{1+cosx}\right)dx=\int e^x\left(\frac{1}{1+cosx}+\frac{sinx}{1+cosx}\right)dx=\int e^x\left(\frac{1}{2cos^2\frac{x}{2}}+\frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}\right)dx=$
$\int e^x\left(\frac{1}{2}sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx=e^x tan\frac{x}{2}+c$

12. $\int\frac{x^3}{x^4+3x^2+2}$ dx (Hint : put x^2=t, and partial fraction, A= -1, B = 2)

= log$\left|\frac{x^2+2}{\sqrt{x^2+1}}\right|$ +C

13. $\int\frac{dx}{2sin^2x+5\,cos^2x}$ (Hint: divide by cos^2x)

Put tan$x$ = t so that sec^2x dx = dt. Then

$$\text{I} = \int\frac{dt}{2t^2+5}=\frac{1}{2}\int\frac{dt}{t^2+\left(\sqrt{\frac{5}{2}}\right)^2}$$

$$= \frac{1}{2}\frac{\sqrt{2}}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}t}{\sqrt{5}}\right)+\text{C}$$

$$= \frac{1}{\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{5}}\right)+\text{C}.$$

14. $\int\frac{e^x}{(1+e^x)(2+e^x)}$ dx (Hint: e^x=t) Ans: log$\left|\frac{1+e^x}{2+e^x}\right|$ +C

15. $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx = e^x \left(\frac{x-1}{x+1}\right)$ +C (Hint : $\int e^x \left(\frac{x^2-1+1+1}{(x+1)^2}\right)$ dx= $\int e^x \left(\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right)$dx

$= \int e^x \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)$dx , f(x)= $\frac{x-1}{x+1}$, f'(x) =$\frac{2}{(x+1)^2}$)

16. $\int \frac{1}{\cos(x-a)\cos(x-b)}\, dx = \frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x+a)}\right|$ +C (Hint : 1 = $\frac{\sin[(x-b)-(x-a))]}{\sin(a-b)}$

17. $\int \frac{e^x}{e^{2x}+6e^x+8}$ dx = ½ log$\left|\frac{e^x+2}{e^x+4}\right|$ +C (Hint :e^x= t)

18. $\int \frac{1}{sinxcos^3x}\, dx$= log$|tanx|$ +1/2tan^2x + C (Hint : In the Denominator divide and multiply by cosx)

19.

$$\int \frac{\sin(x-a)}{\sin(x+a)}dx$$

Let , $(x+a) = t$

$$\int \frac{\sin(t-2a)}{\sin t}dt = \int \frac{\sin t \cos 2a - \cos t \sin 2a}{\sin t}dt = \int (\cos 2a - \cot t \sin 2a)dt$$

$$\therefore \int \frac{\sin(t-2a)}{\sin t}dt = (\cos 2a)t - (\sin 2a)\log|\sin t| + C$$

20. $\int \frac{dx}{x(x^4-1)}$ = 1/4 log$\left|\frac{x^4-1}{x^4}\right|$ +C (Hint : Multiply and divide by x^3, and partial fraction)

ANSWERS OF LONG ANSWER QUESTIONS:

1. $\int \frac{sin^8x-cos^8x}{1-2sin^2xcos^2x}$ dx = $\int \frac{(sin^4x+cos^4x)(sin^4x-cos^4x)}{(sin^2x+cos^2x)^2-2sin^2xcos^2x}$ =

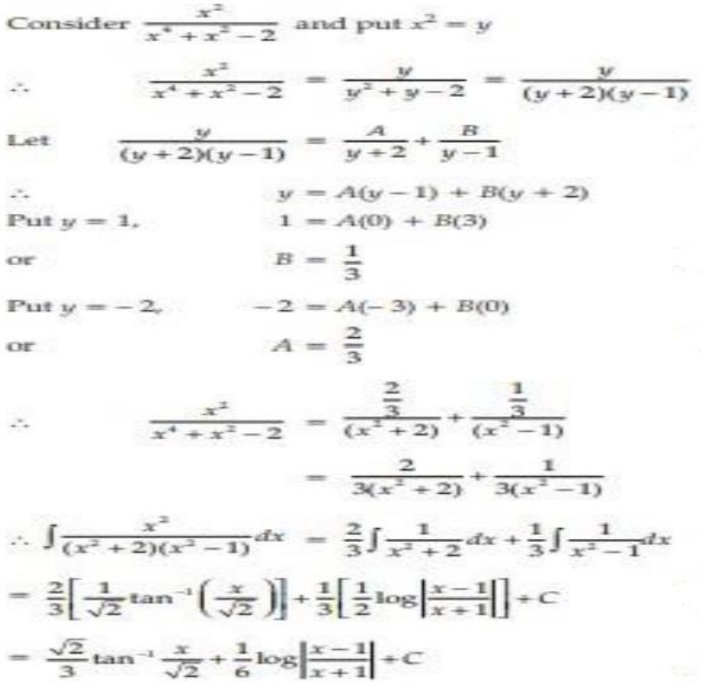
$\int \frac{(sin^4x+cos^4x)(sin^2x+cos^2x)(sin^2x-cos^2x)}{(sin^4x+cos^4x)} = \int -cos\,2x\,dx = -\frac{1}{2}sin2x + C$

2. $\int \frac{x+2}{2x^2+6x+5}\, dx$= 1/4log$|2x^2+6x+5|$ +1/2tan^{-1}(2x+3) +C

(Hint: x +2 = A (4x +6) + B

3. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx= .$-2\sqrt{1-x} + cos^{-1}\sqrt{x} + \sqrt{x-x^2}$ −

4. Find $\int \frac{x^2}{x^4+x^2-2}\ dx$



Consider $\frac{x^2}{x^4+x^2-2}$ and put $x^2 = y$

$$\therefore \quad \frac{x^2}{x^4+x^2-2} = \frac{y}{y^2+y-2} = \frac{y}{(y+2)(y-1)}$$

$$\text{Let} \quad \frac{y}{(y+2)(y-1)} = \frac{A}{y+2} + \frac{B}{y-1}$$

$$\therefore \quad y = A(y-1) + B(y+2)$$

$$\text{Put } y = 1, \quad 1 = A(0) + B(3)$$

$$\text{or} \quad B = \frac{1}{3}$$

$$\text{Put } y = -2, \quad -2 = A(-3) + B(0)$$

$$\text{or} \quad A = \frac{2}{3}$$

$$\therefore \quad \frac{x^2}{x^4+x^2-2} = \frac{\frac{2}{3}}{(x^2+2)} + \frac{\frac{1}{3}}{(x^2-1)}$$

$$= \frac{2}{3(x^2+2)} + \frac{1}{3(x^2-1)}$$

$$\therefore \int \frac{x^2}{(x^2+2)(x^2-1)} dx = \frac{2}{3}\int \frac{1}{x^2+2} dx + \frac{1}{3}\int \frac{1}{x^2-1} dx$$

$$= \frac{2}{3}\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|\right] + C$$

$$= \frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + C$$

5. $\int \frac{2}{(1-x)(1+x^2)} dx$ = -log$|x-1|$ +1/2 log$|1+x^2|$ +tan^{-1}x +C

(Hint : $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

6. $\int \frac{3x-1}{(x+2)^2}$ dx= 3 log$|x+2|$ +$\frac{7}{x+2}$ +C (Hint : $\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$)

7. $\int tan^{-1}\sqrt{\frac{1-x}{1+x}}$ dx = $\frac{1}{2}\left(xcos^{-1}x - \sqrt{1-x^2}\right)$ +C

(Hint : x= cos 2θ, *then by parts*)

8. $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}$ dx =3 (Hint: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

9. $\int_{-1}^{2}|x^3-x|\ dx$= 11/4 (Hint: $\int_{-1}^{0}(x^3-x)\ dx + \int_0^1 -(x^3-x)dx +$
$\int_1^2 (x^3-x)\ dx$)

10. $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}$ dx= $\frac{1}{40}$ log 9

(Hint : sin x-cosx = t, (sinx +cosx)dx = dt

$(-cosx + sinx)^2$=t^2, 1- sin2x =t^2, sin2x = 1-t^2

I =$\int_{-1}^{0} \frac{dt}{25-16t^2}$

= $\frac{1}{40}$ log 9 (After applying limit in 1/40 log$\left|\frac{5+4t}{5-4t}\right|$

11. $\int_0^{\pi} \frac{x\ tanx}{secx+tanx}$dx = $\frac{\pi}{2}(\pi-2)$

(Hint : $\int_0^a f(x)dx$ =$\int_0^a f(a-x)dx$

2 I= $\pi\int_0^{\pi} \frac{tanx}{secx+tanx}$,

I = $\pi/2\int_0^{\pi} \left(\frac{tanx\ (secx-tanx)}{(secx+tanx)(secx-tanx)}\right)dx$

= $\pi/2\int_0^{\pi}\ (secx\ tanx - tan^2\ x)dx$ integrate & applying the limits for getting the answer.

12.$LetI =\ \int_0^{\pi/2} \frac{sin^2x}{sinx+cosx}$ dx & applying $\int_0^a f(x)dx$ =$\int_0^a f(a-x)dx$

2I = $\int_0^{\pi/2} \frac{1}{sinx+cosx}$ dx= $\int_0^{\pi/2} \frac{1}{\sqrt{2}}\left(\frac{dx}{\frac{1}{\sqrt{2}}sinx+\frac{1}{\sqrt{2}}cosx}\right)$

= $\int_0^{\pi/2} \frac{1}{\sqrt{2}}\left(\frac{dx}{\sin\ (x+\frac{\pi}{4})}\right)$dx

=$\int_0^{\pi/2} cosec\ \left(x+\frac{\pi}{4}\right)$dx

=$\frac{1}{\sqrt{2}}\left[log\left|cosec\left(x+\frac{\pi}{4}\right) - \cot\ (x+\pi/4\right|\right]_0^{\frac{\pi}{2}}$

= $\frac{1}{\sqrt{2}}$ log ($\sqrt{2}$ +1) After applying limit

13.$\int \frac{x^4}{(x-1)(x^2+1)}$dx = $\frac{x^2}{2}$ +x +1/2 log $|x-1|$ -1/4 log$|x^2+1|$ -1/2 $tan^{-1}x$ +C

(Hint: degree of numerator is greater than denominator , so divide.

$\frac{x^4}{(x-1)(x^2+1)}$ = (x+1) +$\frac{1}{(x-1)(x^2+1)}$, use partial fraction)

14.$\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx = $\frac{\pi}{8}$ log 2

(Hint: Put x= tan$\theta, to\ get\ \int_0^{\pi/4}$ log$(1+tan\theta)d\theta$ and then applying $\int f(x)dx$ =$\int f(a-x)dx$)

15. $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+\ cos^4x}$ dx= .$\frac{\pi^2}{16}$ (Hint: $\int_0^a f(x)dx$ =$\int_0^a f(a-x)dx$ and then divide numerator and denominator by cos^4x and put tan^2x = t)

ANSWERS OF CASE STUDY:

CASE STUDY 1:

(i)(A)1 (ii) (B)1 (iii) (B)2 (iv) (B)1

CASE STUDY 2:

(i)$(A)\log|1+\log x|+C$ (ii) (B)$\frac{(\sin^{-1}x)^3}{3}+C$ (iii) (C)$-2\cos\sqrt{x}+C$

(iv) (B)$2(e-1)$

CASE STUDY 3:

(i) B) $-\cos(x^2+1)+C$ (ii) B) $-cos(\tan^{-1}x)+C$, (iii) A) $\frac{(\sin^{-1}x)^2}{2}$ + C

(iv) B) $\frac{1}{1+cosx}$ + C

CASE STUDY 4:

(i) D) $\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{\tan^{-1}x}{2}+C,$ (ii) A) x log x - x + C , (iii) B)x tan x - log |sec x| + C

(iv) A) $\frac{x^2}{2}$ log 2x $-\frac{x^2}{4}$ + C

CASE STUDY 5:

(i) C)$e^x sinx+C$ (ii) A) $\frac{e^x}{x}+C$ (iii) C) $-e^x cot\frac{x}{2}+C$ (iv)A) $\frac{x-1}{x+1}e^x+C$

****

CHAPTER 8

APPLICATIONS OF INTEGRALS

LEARNING OUTCOMES

- Area under the simple curves
- Area of the region bounded by a curve and a line

Standard form of curves

- **Parabolas**

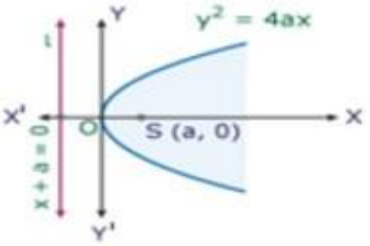


Fig.i

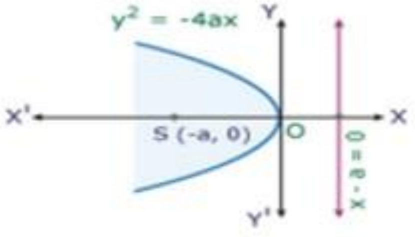


Fig.ii

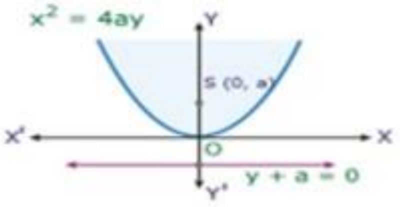


Fig.iii

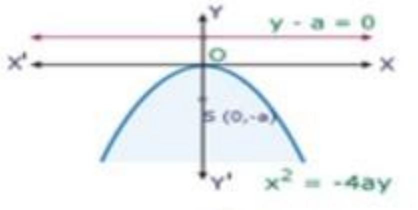
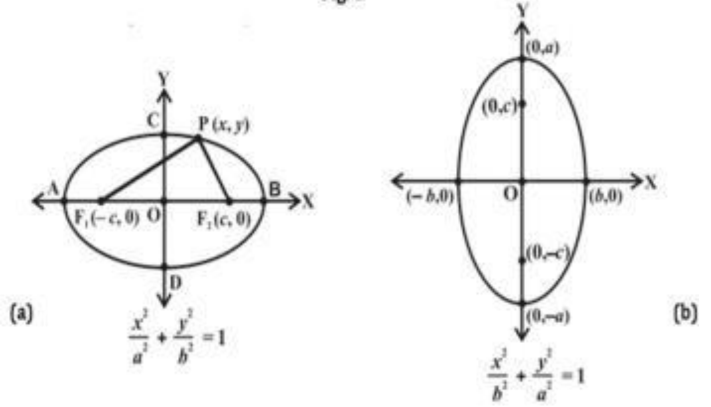


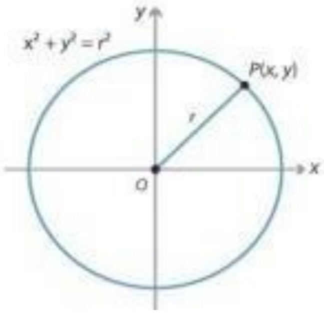
Fig.iv

ELLIPSE

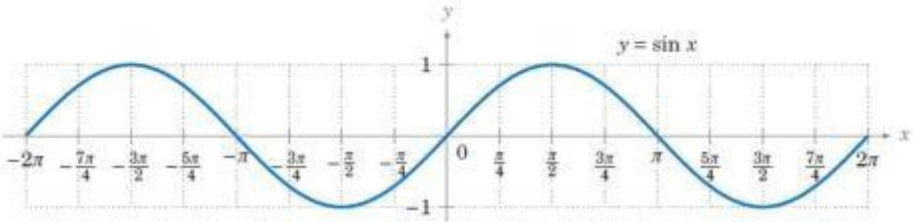
Fig. 5



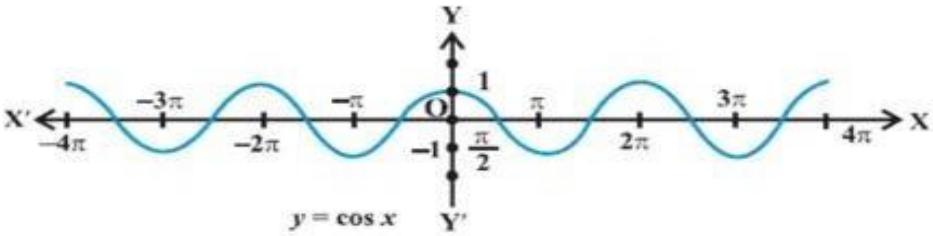
CIRCLE



- <u>Sine curve</u>

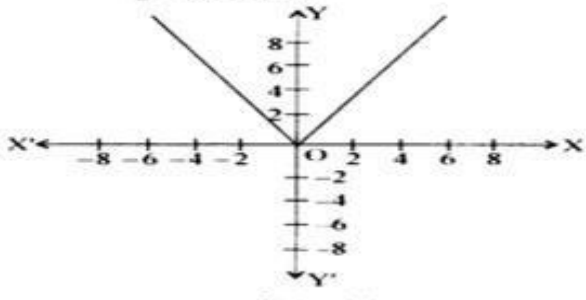


COSINE CURVE



- <u>Modulus function</u>

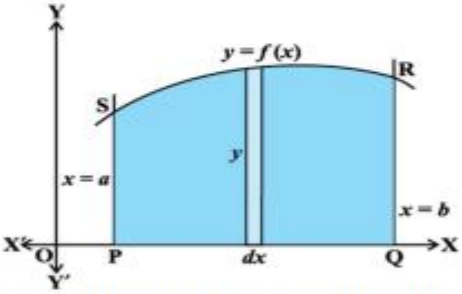
$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



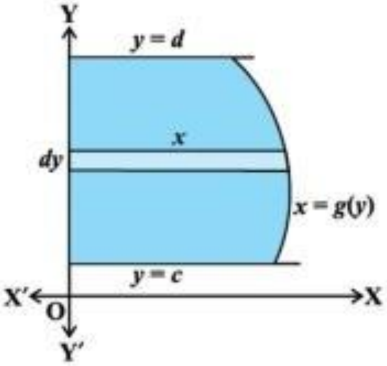
$f(x) = |x|$

Area under simple curves

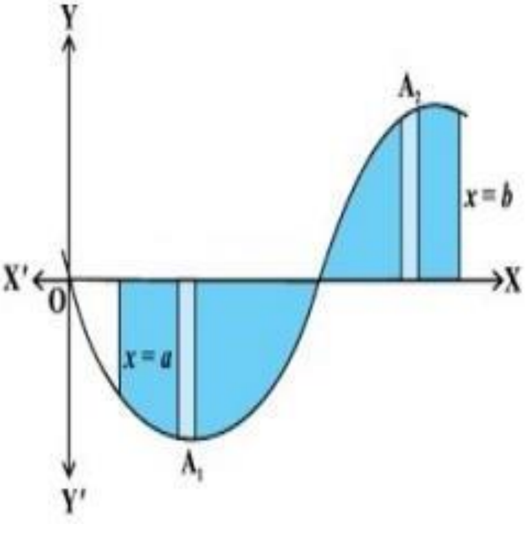
- The area A of the region bounded by the curve y=f(x), x-axis and the line x=a and x=b is given by A $= \int_a^b y\,dx = \int_a^b f(x)\,dx$



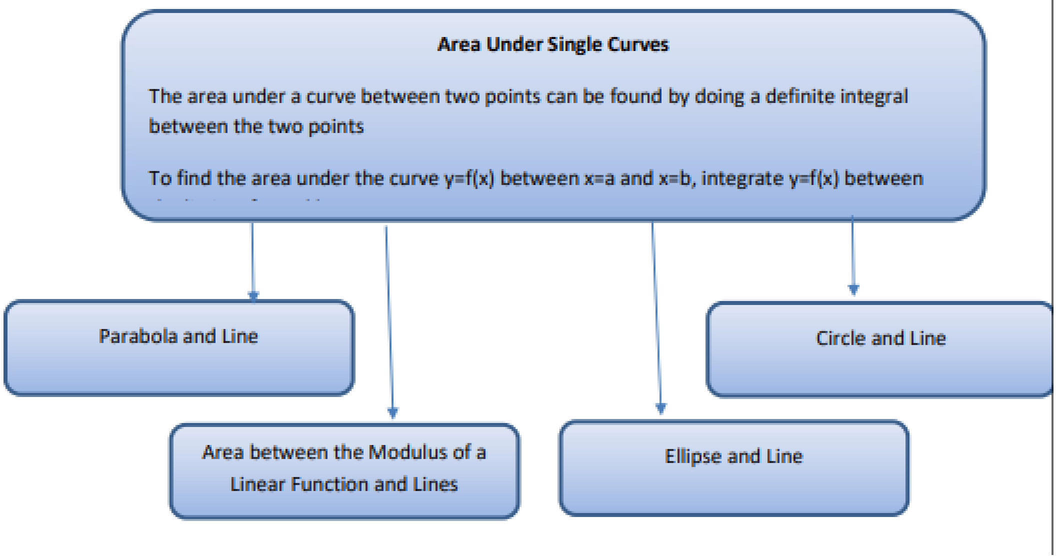
- The area A of the region bounded by the curve x=f(y), y-axis and the line y=c and y=d is given By A $= \int_c^d x\,dy = \int_c^d f(y)\,dy$



- The area A of the region bounded by the curve y=f(x), x-axis and the line x=a and x=b is given by A = $|A_1| + A_2$



MIND MAP



Area Under Single Curves

The area under a curve between two points can be found by doing a definite integral between the two points

To find the area under the curve y=f(x) between x=a and x=b, integrate y=f(x) between

Parabola and Line

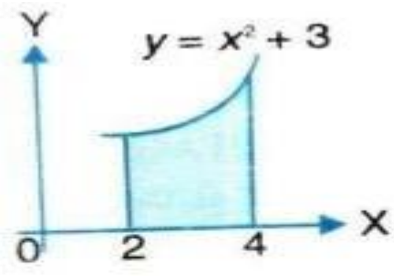
Area between the Modulus of a Linear Function and Lines

Ellipse and Line

Circle and Line

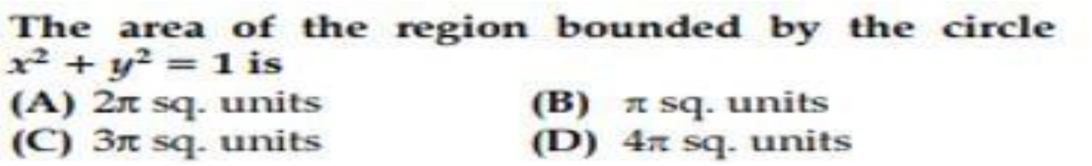
MULTIPLE CHOICE QUESTIONS :

Q.1. The expression for finding the area of the shaded region is -------------



A) $\int_0^4 (x^2 + 3)\, dx$ (B) $\int_2^4 (x^2 + 3)dx$ (C) $\int_2^4 (x^2 + 4)\, dx$ (D) $\int_0^4 (x^2 - 3)dx$

Q.2.

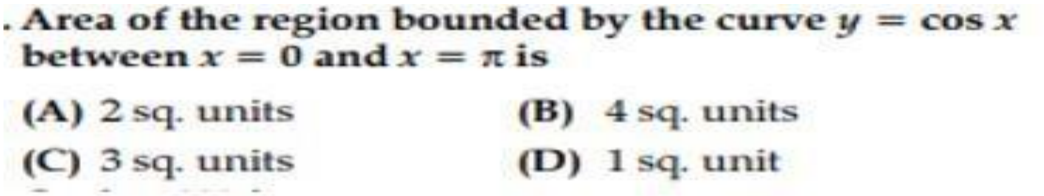


The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(A) 2π sq. units **(B)** π sq. units

(C) 3π sq. units **(D)** 4π sq. units

Q.3

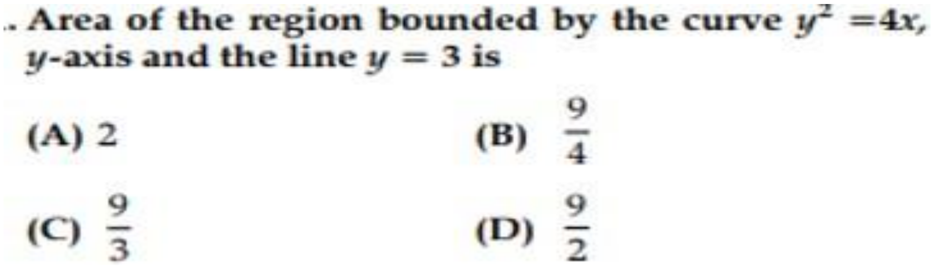


. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(A) 2 sq. units **(B)** 4 sq. units

(C) 3 sq. units **(D)** 1 sq. unit

Q.4

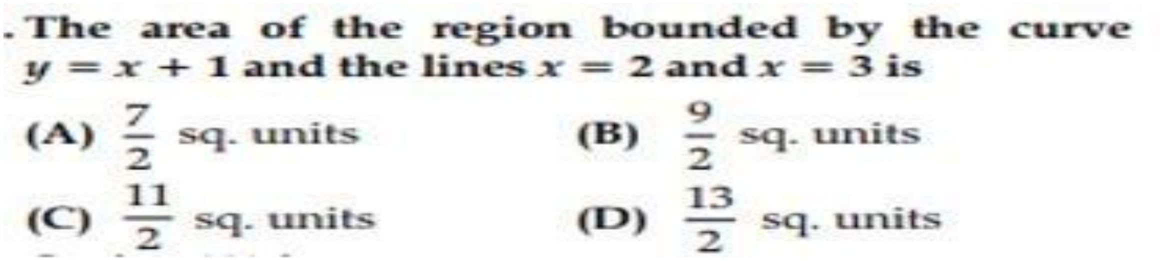


.. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 **(B)** $\frac{9}{4}$

(C) $\frac{9}{3}$ **(D)** $\frac{9}{2}$

Q.5



. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is

(A) $\frac{7}{2}$ sq. units **(B)** $\frac{9}{2}$ sq. units

(C) $\frac{11}{2}$ sq. units **(D)** $\frac{13}{2}$ sq. units

SHORT ANSWER QUESTIONS:

Q.6 Find the area of the region bounded by the ellipse$\frac{x^2}{25}+\frac{y^2}{16} = 1$

Q.7 Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line x + y = 2

Q.8 Find the area lying between the curves $y^2 = 4ax$ and y=2x

Q.9.Sketch the graph of y=|x + 3| and evaluate $\int_{-6}^{0}|x + 3|\, dx$

Q.10 Find the area bounded by the curve y= x^3 ,the x-axis and the ordinates x=-2 and x=1

Q.11.Find the area of the region bounded by the line y=3x+2 , the x-axis and the ordinates $x = -1 \; and \; x = 1$.

Q.12 Find the area under the given curve y=x^2 , x=-1, x=1 and x-axis

Q.13 Find the area bounded by the curve y= sin x between x=0 and x=2π

Q.14 Compute the area of the region bounded by the curve y= 2^x and the lines x=1 to x=3

Q.15Find the area bounded by the curve y= Cot x, X-axis and the lines $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{4}$

LONG ANSWER QUESTIONS:

Q. 16. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Q.17 Find the area of the region

$$\{(x, y) : x^2 + y^2 \leq 1 \leq (x + y)\}.$$

Q. 18. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area using integration

Q.19

Using integration find the area of the region:
$\{(x, y) : 0 \leq y \leq x^2,\ 0 \leq y \leq x, 0 \leq x \leq 2\}$

Q .20

. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $3x + 4y = 12$.

Q.21 Find the area bounded by the curves $y=\sqrt{x}$, 2y+3=x and x-axis

Q.22 Find the area of the region bounded by the curves $x^2 + y^2 = 4$, y= $\sqrt{3}\,x$ and x -axis in the first quadrant

Q.23 Find the area of the region bounded by the parabola y^2 =x and the line 2y= x.

Q.24 Find the area of the region in the first quadrant enclosed by the Y-axis, the line y=x and the circle

$x^2 + y^2 = 32$, using integration

Q.25 Using integration, find the area of the region bounded by the line x-y+2 = 0, the curve $x=\sqrt{y}$ and the Y -axis

CASE STUDY QUESTIONS :

1. Consider the curve $x^2 + y^2 = 16$, and the line $y = x$ in the first quadrant.

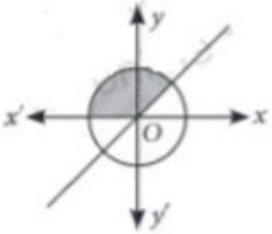
Based on the above information, answer the following questions.

(i) Point of intersection of both the given curves is

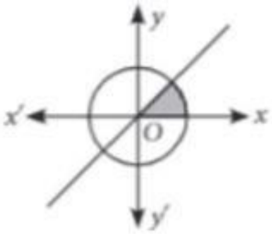
(a) (0,4) (b) $(0,2\sqrt{2})$ (c) $(2\sqrt{2}, 2\sqrt{2})$ (d) $(2\sqrt{2},4)$

(ii) Which of the following shaded portion represent the area bounded by the giventwo curves

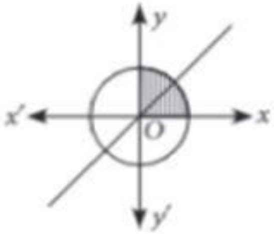
a)



(b)



(c)



d) none of these

(iii) The value of the integral $\int_0^{2\sqrt{2}} x\,dx$ is

(a) 0 (b) 1 (c) 2 (d) 4

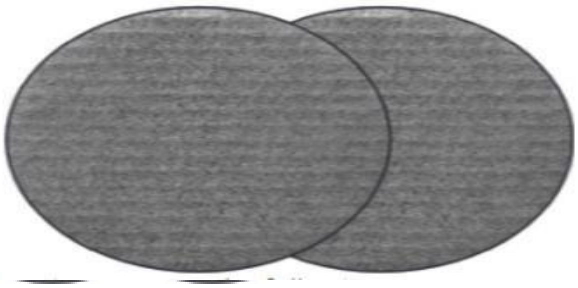
(iv) The value of the integral $\int_{2\sqrt{2}}^{4} \sqrt{16 - x^2}$ is

(a) $2(\pi - 2)$ (b) $2((\pi - 8)$ (c) $4(\pi - 2)$ (d) $4(\pi + 2)$

(v) Area bounded by the two given curves is

(a) 3π sq.units (b) $\frac{\pi}{2}$ sq units (c) π sq units (d) $2\pi\ sq\ units$

Q.2 Ajay cut two circular pieces of card boards and placed one upon other shown in the figure.The figure represents the equation $(x-1)^2 + y^2 = 1$,while other represents equation $x^2 + y^2 = 1$,

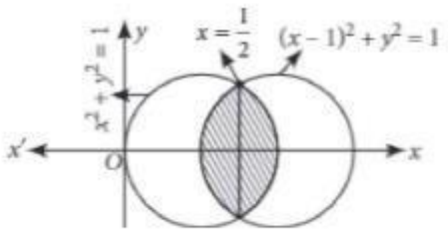


Based on the above information , answer the following

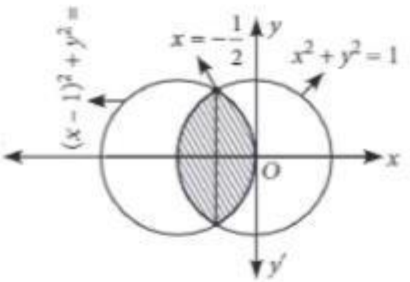
(i) Both the circular pieces of card board meet each other at

(a) x = 1 (b) $x = \frac{1}{2}$ (c) $x = \frac{1}{3}$ (d) $x = \frac{1}{4}$

(ii) Graph of given two curves can be drawn as

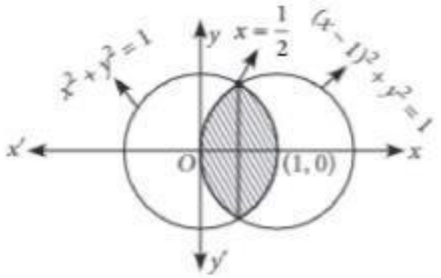


(a)



(b)

(c)



(d) None of these

(iii) Value of $\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx$ is

(a) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$ (b) $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$ (c) $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

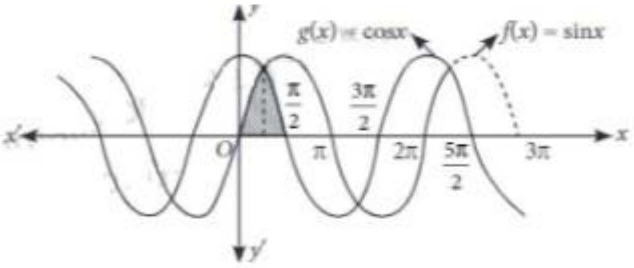
(iv) Value of $\int_{1/2}^{1} \sqrt{1-x^2}$ dx is

(a) $\frac{\pi}{2}+\frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$ (c) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ (d) $\frac{\pi}{2}-\frac{\sqrt{3}}{4}$

(v) Area of hidden portion of lower circle is

(a) $(\frac{2\pi}{3}+\frac{\sqrt{3}}{2})$ sq units (b) $(\frac{\pi}{3}-\frac{\sqrt{3}}{8})$ sq units (c) $(\frac{\pi}{3}+\frac{\sqrt{3}}{8})$ sq units (d) $(\frac{2\pi}{3}-\frac{\sqrt{3}}{2})$ sq units

Q.3 Graphs of two functions f(x) = sin x and g(x) = cos x given below



Based on the above informtion,answer the following questions

(i) In [0,π] the curves f(x) = sin x and g(x) = cos x intersect at x =

(a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) π

(ii) Value of $\int_{0}^{\pi/4} sinx$ dx is

(a) $1-\frac{1}{\sqrt{2}}$ (b) $1+\frac{1}{\sqrt{2}}$ (c) $2-\frac{1}{\sqrt{2}}$ (d) $2+\frac{1}{\sqrt{2}}$

(iii) Value of $\int_{\pi/4}^{\pi/2} \cos x$ dx is

(a) $1-\frac{1}{\sqrt{2}}$ (b) $1+\frac{1}{\sqrt{2}}$ (c) $2-\sqrt{2}$ (d) $2+\sqrt{2}$

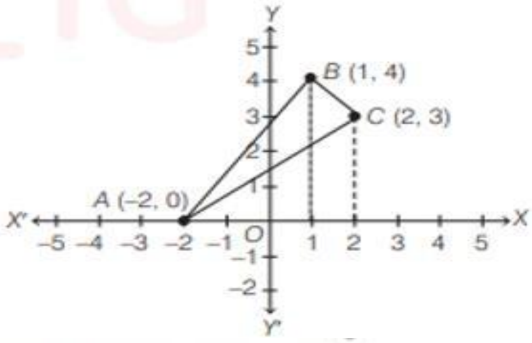
(iv) Value of $\int_{0}^{\pi} \sin x$ dx is

(a) 0 (b) 1 (c) 2 (d) -2

(v) Value of $\int_{0}^{\pi/2} \sin x$ dx isDIFFERE

(a) 0 (b) 1 (c) 3 (d) 4

Q.4 Location of three branches of a bank is represented by the three points A(-2,0) ,B(1,4) and C(2,3) as shown in figure .Point D is (2,0) .



(i) Equation of line AB is

(a) $y = \frac{4}{3}(x + 2)$ (b) $y = \frac{4}{3}(x + 1)$

(c) $y = \frac{4}{5}(x + 2)$ (d) $y = \frac{4}{5}(x + 1)$

(ii) Equation of line BC is

(a) $y = x + 5$ (b) $y = -x + 5$

(c) $y = x + 4$ (d) $y = -x + 4$

(iii) Area of region $ABCD$ is

(a) 19 sq units (b) $\frac{19}{2}$ sq units

(c) 17 sq units (d) 6 sq units

(iv) Area of ΔADC is

(a) 3 sq units (b) 4 sq units

(c) 6 sq units (d) 5 sq units

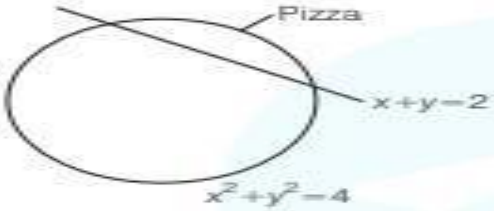
(v) Area of ΔABC is

(a) 7 sq units (b) $\frac{3}{2}$ sq units

(c) 5 sq units (d) $\frac{7}{2}$ sq units

Q.5

Aman was celebrating his birthday with his friends. He ordered a pizza. He cut the pizza with a knife. Pizza was circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represents a straight line given by $x + y = 2$.



Based on the above information, answer the followwing questions.

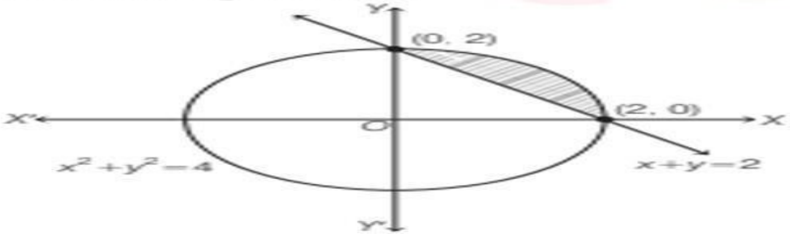
(i) The points of intersection of the edge of knife (line) and pizza shown in figure is (are)

(a) (0, 2) and (2, 0) (b) (0, 1) and (1, 0)

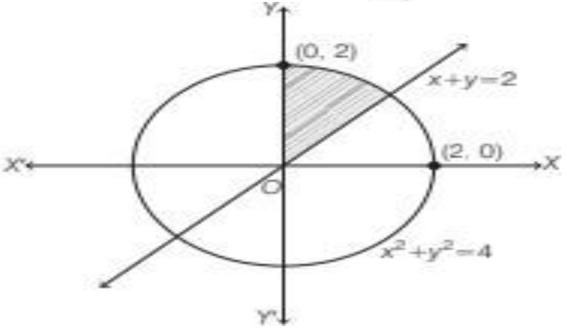
(c) (1, 2) and (2, 1) (d) (3, 1) and (1, 3)

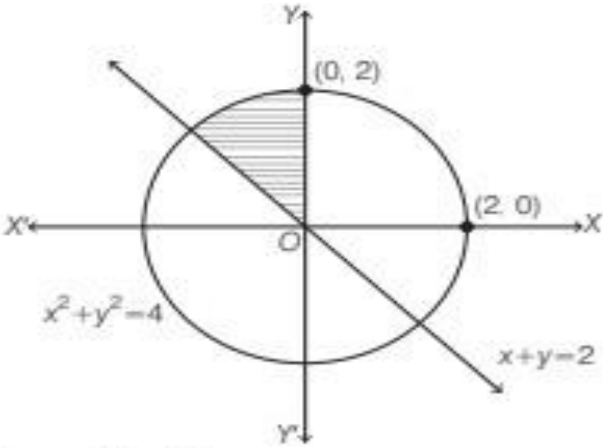
(ii) Which of the following shaded portion represent the smaller area bounded by pizza and edge of knife in first quadrant?

(a)



(b)



(c) 

(d) None of the above

(iii) The area bounded by the sharp edge of knife with the coordinate axes is

(a) $\frac{1}{4}$ sq unit (b) $\frac{1}{2}$ sq unit

(c) 2 sq units (d) 3 sq units

(iv) Area of each slice of pizza, when Aman cut the pizza into 4 equal pieces is

(a) $\frac{\pi}{2}$ sq units (b) π sq units

(c) $\frac{\pi}{3}$ sq units (d) 2π sq units

(v) Area of whole pizza is

(a) 4π sq units (b) 3π sq units

(c) π sq units (d) $\frac{\pi}{2}$ sq units

ANSWERS:

MCQ ANSWERS:

Q.1 (B),

Q.2 (B)

Q.3 (A)

Q.4 (B)

Q.5(A)

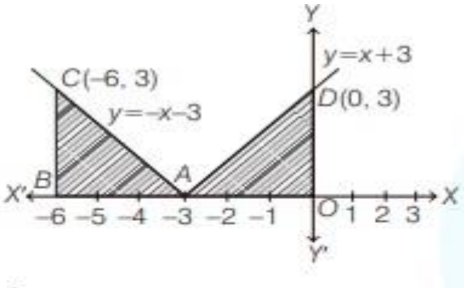
SHORT ANSWER QUESTIONS :

Q.6 20 π sq units

Q.7 ($\pi - 2$) sq units

Q.8 $\frac{1}{3} sq\ units$

Q.9



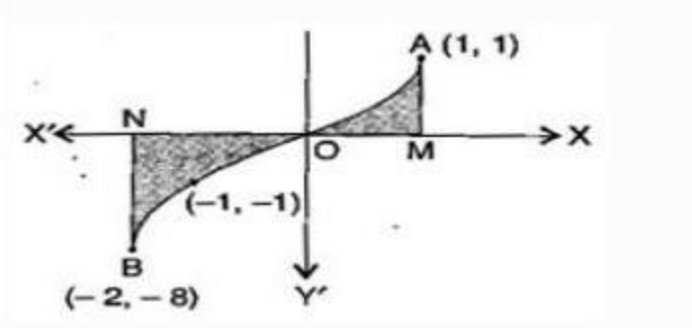
∴ Required area

$= $ Area of region ABC + Area of region OAD

$= \int_{-6}^{-3} |x + 3|\, dx + \int_{-3}^{0} |x + 3|\, dx$

Integrating and applying limits to get the area shaded is equal to 9 sq.units

Q.10.



$$= \left|\int_{-2}^{0} x^3\, dx\right| + \left|\int_{0}^{1} x^3\, dx\right|$$

$$= \left|\left(\frac{x^4}{4}\right)_{-2}^{0}\right| + \left|\left(\frac{x^4}{4}\right)_{0}^{1}\right|$$

$$= \left|0 - \frac{16}{4}\right| + \left|\frac{1}{4} - 0\right|$$

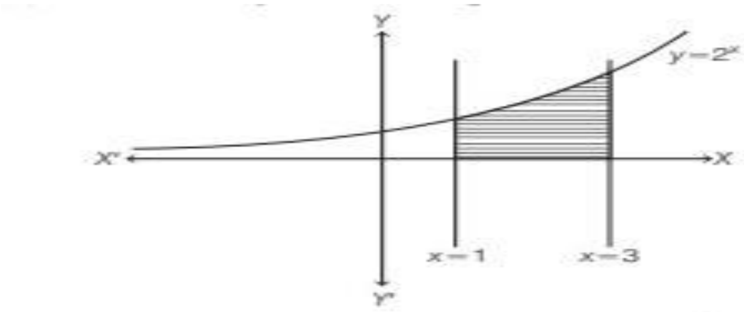
$$= 4 + \frac{1}{4} = \frac{17}{4} \text{ sq. units.}$$

Q.11 $\frac{13}{3}\, sq.units$

Q.12 $\frac{2}{3}\, sq.units$

Q.13 $4\, sq.units$

Q. 14



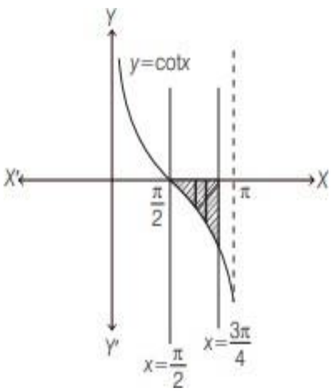
$$\therefore \text{Area of bounded region} = \int_1^3 2^x\, dx = \left[\frac{2^x}{\log 2}\right]_1^3$$

$$= \frac{1}{\log 2}[2^3 - 2^1] = \frac{1}{\log 2}(8 - 2)$$

$$= \frac{6}{\log 2} \text{ sq units}$$

Q.15

$$\therefore \text{Area of bounded region} = \left|\int_{\pi/2}^{3\pi/4} y\,dx\right|$$
$$= \left|\int_{\pi/2}^{3\pi/4} \cot x\,dx\right|$$
$$= \left|[\log|\sin x|]_{\pi/2}^{3\pi/4}\right|$$
$$= \left|\log\left|\sin\frac{3\pi}{4}\right| - \log\left|\sin\frac{\pi}{2}\right|\right|$$
$$= \left|\log\frac{1}{\sqrt{2}} - \log 1\right|$$
$$= |\log 1 - \log\sqrt{2} - \log 1|$$
$$= |-\log\sqrt{2}| = \log\sqrt{2} \text{ sq units}$$



ANSWERS OF LONG ANSWER QUESTIONS:

16. 27 sq.units

17. $\frac{\pi}{4} - \frac{1}{2} su.units$

18. $\frac{5\pi}{4} - \frac{1}{2} sq.units$

19 .$\frac{11}{6} sq.units$

20. $(3\pi - 6)sq.units$

21. 9 sq.units

22. $\frac{2\pi}{3} sq.units$

23.$\frac{4}{3} sq.units$

24. $4\pi sq.units$

25. .$\frac{10}{3}$sq.units

ANSWERS OF CASESTUDY QUESTIONS:

Q.1 (i) c (ii) b (iii) d (iv) a (v) d

Q.2 (i) b (ii) c (iii) a (iv) c (v) d

Q.3 (i) c (ii)a (iii) a (iv)c (v) b

Q.4 (i) a (ii) b (iii)b (iv) c (v)d

Q.5 (i) a (ii)a (iii) c (iv) b (v) a

CHAPTER 9

DIFFERENTIAL EQUATIONS

SYLLABUS

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}$+ py = q, where p and q are functions of x or constants.

$\frac{dx}{dy}$+ px = q, where p and q are functions of y or constants.

Points To Remember

An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation. If there is only one independent variable, then we call it an ordinary differential equation.

Order of a Differential Equation Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

Degree of a Differential Equation Degree of a differential equation, when it is a polynomial equation in derivatives, is defined as the highest power (exponent) of the highest order derivatives involved in the given differential equation.

Solution of a Differential Equation

A function which satisfies the given differential equation is called its solution.

General and Particular Solution of a Differential Equation

The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called a particular solution.

Methods of solving first order ,first degree Differential Equations

1. Differential equations with variable separable Variable separable method is used to solve an equation in which variables can be separated completely i.e. terms containing y should remain with dy and terms containing x should remain with dx

eg: y dx =x dy can be solved as $\frac{dx}{x} = \frac{dy}{y}$

Integrating both sides logx = logy + log c

$\frac{x}{y}$ = c

x = cy is the solution

2. Homogeneous differential equations

a) A differential equation which can be expressed in the form $\frac{dy}{dx}$ = f(x,y) where f(x,y) is a homogeneous function of degree zero is called homogeneous differential equation

Example : $(x^2$ +xy) dy = $(x^2$ $+y^2$) dx

To solve this, we substitute y=vx and $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$

b) A differential equation which can be expressed in the form $\frac{dx}{dy}$ = f(x,y) where f(x,y) is a homogeneous function of degree zero is called homogeneous differential equation

Example

$\left(e^{\frac{x}{y}}\right)$ dx + $e^{\frac{x}{y}}\left(\frac{x}{y}\right)$ dy = 0

To solve this, we substitute x = vy and $\frac{dx}{dy}$ = v + $y\frac{dv}{dy}$

3. Linear differential equations

a) A differential equation of the form $\frac{dy}{dx}$ + Py = Q where P and Q are constants or functions of x only is called first order linear differential equation

Its solution is given as y $e^{\int Pdx} = \int Qe^{\int Pdx}$ dx + c

Example : $\frac{dy}{dx}$ + 3y =2x has solution

y $e^{\int 3dx}$ = $\int 2xe^{\int 3dx}$ dx + c

b) A differential equation of the form $\frac{dx}{dy}$ + Px = Q where P and Q are constants or functions of y only is called first order linear differential equation

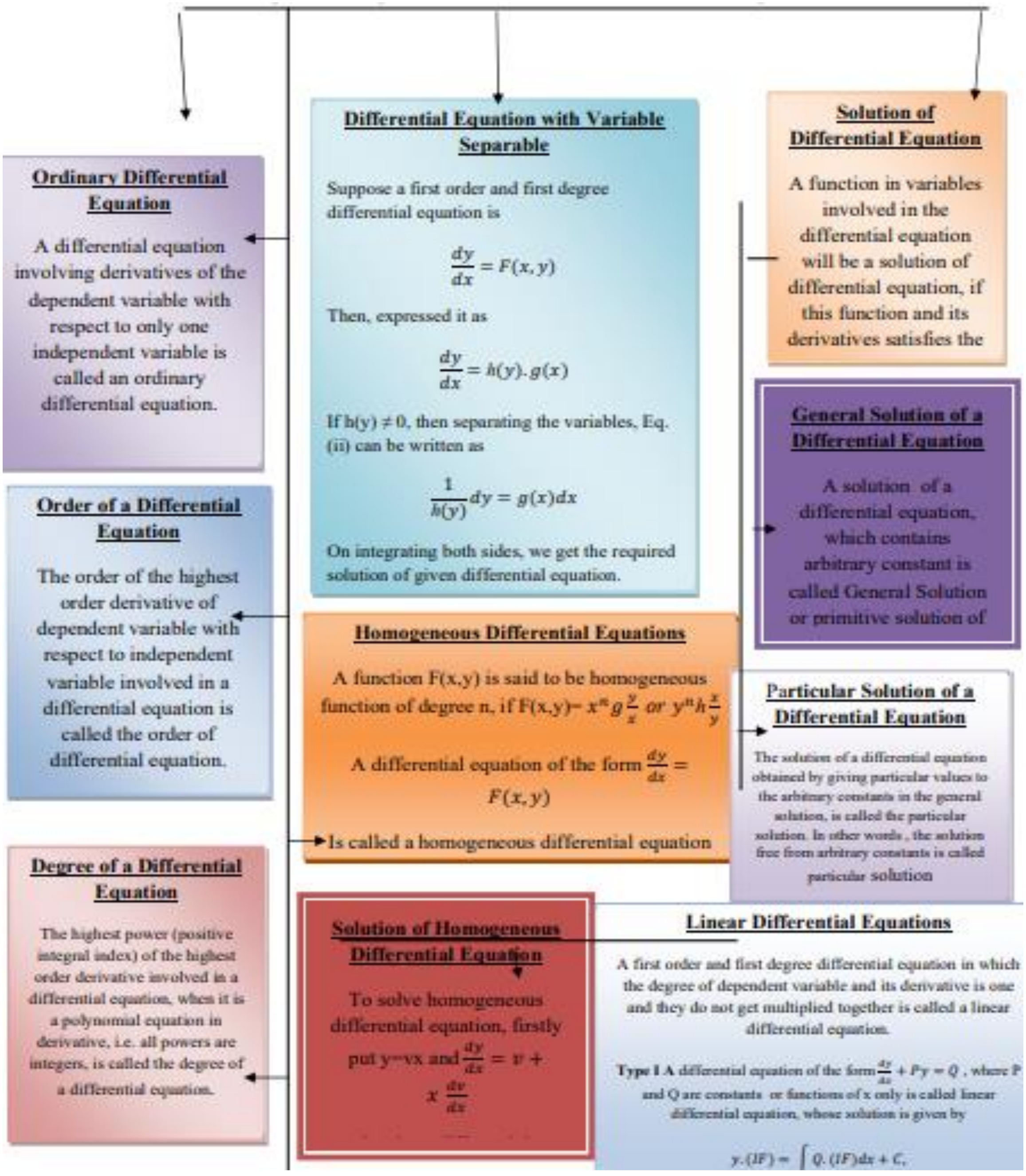
Its solution is given as x $e^{\int Pdy} = \int Qe^{\int Pdy}$ dy + c

Example : $\frac{dx}{dy}$ - $\frac{x}{y}$ = 2y has solution

x $e^{\int \frac{-1}{y}\,dy}$ = $\int 2y\, e^{\int \frac{-1}{y}\,dy}$ dy + c

MIND MAPPING

An equation containing an independent variable, dependent variable and derivative of dependent variable with respect to independent variable is called a differential equation.



Ordinary Differential Equation

A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.

Order of a Differential Equation

The order of the highest order derivative of dependent variable with respect to independent variable involved in a differential equation is called the order of differential equation.

Degree of a Differential Equation

The highest power (positive integral index) of the highest order derivative involved in a differential equation, when it is a polynomial equation in derivative, i.e. all powers are integers, is called the degree of a differential equation.

Differential Equation with Variable Separable

Suppose a first order and first degree differential equation is

$$\frac{dy}{dx} = F(x, y)$$

Then, expressed it as

$$\frac{dy}{dx} = h(y).g(x)$$

If h(y) ≠ 0, then separating the variables, Eq. (ii) can be written as

$$\frac{1}{h(y)}dy = g(x)dx$$

On integrating both sides, we get the required solution of given differential equation.

Homogeneous Differential Equations

A function F(x,y) is said to be homogeneous function of degree n, if F(x,y)= $x^n g\frac{y}{x}$ or $y^n h\frac{x}{y}$

A differential equation of the form $\frac{dy}{dx} = F(x, y)$

Is called a homogeneous differential equation

Solution of Homogeneous Differential Equation

To solve homogeneous differential equation, firstly put y=vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Solution of Differential Equation

A function in variables involved in the differential equation will be a solution of differential equation, if this function and its derivatives satisfies the

General Solution of a Differential Equation

A solution of a differential equation, which contains arbitrary constant is called General Solution or primitive solution of

Particular Solution of a Differential Equation

The solution of a differential equation obtained by giving particular values to the arbitrary constants in the general solution, is called the particular solution. In other words , the solution free from arbitrary constants is called particular solution

Linear Differential Equations

A first order and first degree differential equation in which the degree of dependent variable and its derivative is one and they do not get multiplied together is called a linear differential equation.

Type I A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is called linear differential equation, whose solution is given by

$$y.(IF) = \int Q.(IF)dx + C,$$

MULTIPLE CHOICE QUESTIONS

1) The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\ \right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(A) 4 (B) $\frac{3}{2}$ (C) not defined (D) 2

2) Solution of the differential equation tany $sec^2x\ \ dx$ + tanx $sec^2y\ dy$ = 0 is

(A) tanx + tany = k (B) tanx – tany = k

(C) $\frac{tan\ x}{tan\ y} = k$ (D) tanx . tany = k

3) The solution of $\frac{dy}{dx} + y = e^{-x}, y\ (0) = 0$ is :

(A) y = e^x (x – 1) (B) y = xe^{-x}

(C) y = xe^{-x} + 1 (D) y = (x + 1)e^{-x}

4)Integrating factor of the differential equation $\frac{dy}{dx}$ + y tan x – sec x = 0 is:

(A) $cosx$ (B) $secx$ (C) e^{cosx} (D) e^{secx}

5) The solution of the differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is :

(A) y (1 + x^2) = c + $\tan^{-1} x$ (B) $-\frac{y}{1+x^2} = c + \tan^{-1} x$

(C) y log (1 + x^2) = c + $\tan^{-1} x$ (D) y (1 + x^2) = c + $\sin^{-1} x$

SHORT ANSWER QUESTIONS

1)Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

2) Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5

Find the value of x when y = 3

3) Solve the differential equation (x^2 – 1) $\frac{dy}{dx}$ + 2xy = $\frac{1}{(x^2-1)}$

4) Solve the differential equation $\frac{dy}{dx}$ + 2xy = y

5) Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$

6) Solve the differential equation$\frac{dy}{dx} + 1 = e^{x+y}$

7) Solve: ydx – xdy = x^2 ydx

8) Solve the differential equation$\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when y = 0, x = 0.

9)Find the general solution of (x + $2y^3$) $\frac{dy}{dx}$ = y

10) If y(x) is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -cosx\ and\ y\ (0)\ =$
$1, then\ find\ the\ value\ of\ y\ \left(\frac{\pi}{2}\right)$

11) Find the equation of a curve passing through origin and satisfying the differential equation (1 + x^2) $\frac{dy}{dx} + 2xy\ =\ 4x^2$

12) Solve : $x^2\ \frac{dy}{dx}\ = x^2\ + xy\ + y^2$

13) Find the general solution of $y^2\ dx\ + (x^2 - xy + y^2)\ dy = 0$

14) Solve : (x + y) (dx – dy) = dx + dy

15) Solve : 2 (y + 3) – xy $\frac{dy}{dx}$ = 0, given that y (1) = – 2

CASE STUDY QUESTIONS

1) A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat was 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation:$\frac{dT}{dt} \propto (T - 70)$, where 70°F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ where k is a constant of proportion, time of death is calculated.

1) State the degree of the above given differential equation.

2) Which method of solving a differential equation helped in calculation of the time of death?

a) Variable separable method

b) Solving Homogeneous differential equation

c) Solving Linear differential equation

d) all of the above

3) If the temperature was measured 2 hours after 11.30pm, will the time of death change? (Yes/No)

4) The solution of the differential equation $\frac{dT}{dt} = k(\mathrm{T} - 70)$ is given by,

a) log | T – 70| = kt + C b) log | T – 70| = log |kt |+ C

c) T – 70 = kt + C d) T – 70 = kt C

5) If t = 0 when T is 72, then the value of c is

a) -2 b) 0 c) 2 d) Log 2

ANSWERS:

1) Degree is 1

2) (a) Variable separable method

3) No

4) (a) log | T – 70| = kt + C

5) (d)log 2

2) Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx}$ = K (50 − y) where x denotes the number of weeks and y the number of children who have been given the drops.

1. State the order of the above given differential equation.

2. Which method of solving a differential equation can be used to solve $\frac{dy}{dx}$ = k (50 − y).?

a) Variable separable method

b) Solving Homogeneous differential equation

c) Solving Linear differential equation

d) All of the above

3. The solution of the differential equation $\frac{dy}{dx}$= k (50 − y) is given by,

a) log | 50 – y| = kx + C b) - log | 50 – y| = kx + C

c) log | 50 – y| = log| kx |+ C d) 50 – y = kx + C

4. The value of c in the particular solution given that y(0)=0 and k = 0.049 is :

a) log 50 b) log $\frac{1}{50}$ c) 50 d) -50

5. Which of the following solutions may be used to find the number of children who have been given the polio drops?

a) y = 50 – e^{kx} b) y = 50 − e^{-kx} c) y = 50 (1 – e^{-k}) d) y = 50 (e^{kx}– 1)

ANSWERS:

1. Order is 1

2. (a) Variable separable method

3. (b) - log | 50 – y| = kx + C

4. (b) log $\frac{1}{50}$

5. (c) y = 50 ((1 – e^{-k})

3.

A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50°F. At any time t the thermometer reading be T°F and the outside temperature be S°F.

Based on the above information, answer the following questions.

1. Ifλ is positive constant of proportionality, then $\frac{dT}{dt}$ is
   a) λ (T– S)
   b) λ (T+ S)
   c) λTS
   d) –λ (T– S)

2. The value of T(5) is
   a) 30°F
   b) 40°F
   c) 50°F
   d) 60°F

3. The value of T(10) is
   a) 50°F
   b) 60°F
   c) 80°F
   d) 90°F

4. Find the general solution of differential equation formed in given situation.
   a) $log\, T = St + c$
   b) $log\, (T - S) = -\lambda t + c$
   c) $log\, S = tT + c$
   d) $log\, (T + S) = \lambda t + c$

5. Find the value of constant of integration c in the solution of differential equation formed in given situation.

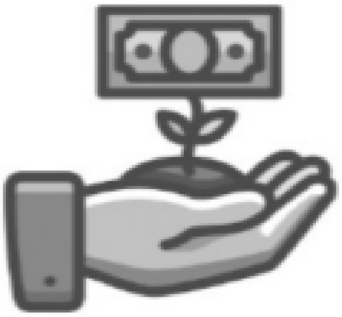
   a) log (60 - S)
   b) log (80 + S)
   c) log (80 - S)
   d) log (60 + 5)

ANSWERS:

1. (d) $-\lambda\,(T-S)$
2. (d) 60°F
3. (a)50°F
4. (b)$log\ S = tT + c$
5. (c) log (80 - S)

4) It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.

Based on the above information, answer the following questions.



1. Find the value of $\frac{dP}{dt}$
   - a) $\frac{Pr}{1000}$
   - b) $\frac{Pr}{100}$
   - c) $\frac{Pr}{10}$
   - d) Pr

2. If P_0 be the initial principal, then find the solution of differential equation formed in given situation.
   - a) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$
   - b) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{10}$
   - c) $\log\left(\frac{P}{P_0}\right) = rt$
   - d) $\log\left(\frac{P}{P_0}\right) = 100rt$

3. If the interest is compounded continuously at 5% per annum, in how many years will ₹100 double itself?
   - a) 12.728 years
   - b) 14.789 years
   - c) 13.862 years
   - d) 15.872 years

4. At what interest rate will ₹100 double itself in 10 years? ($log_e 2$ = 0.6931).
    - a) 9.66%
    - b) 8.239%
    - c) 7.341%
    - d) 6.931%

5. How much will ₹1000 be worth at 5% interest after 10 years? ($e^{0.5}$= 1.648)
    - a) ₹1648
    - b) ₹1500
    - c) ₹1664
    - d) ₹1572

ANSWERS

1. (b)$\frac{Pr}{100}$
2. (a) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$
3. (c) 13.862 years
4. (d)6.931%
5. (a)₹1648

5) If the equation is of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}or\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, where $f(x,y), g(x,y)$ are homogeneous functions of same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ so that the dependent variable y is changed toanother variable v and then apply variable separable method.

Based on the above information, answer the following questions.

1. The general solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is:
   a) $\tan^{-1}\frac{x}{y} = \log|x| + c$
   b) $\tan^{-1}\frac{y}{x} = \log|x| + c$
   c) $y = x\log|x| + c$
   d) $x = y\log|y| + c$

2. Solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is :
   a) $x^3 + y^2 = cx^2$
   b) $\frac{x^2}{2} + \frac{y^3}{3} = y^2 + c$
   c) $x^2 + y^3 = cx^2$
   d) $x^2 + y^2 = cx^3$

3. Solution of the differential equation $(x^2 + 3xy + y^2)dx - x^2dy = 0$ is
   a) $\frac{x+y}{x} - \log x = c$
   b) $\frac{x+y}{x} + \log x = c$
   c) $\frac{x}{x+y} - \log x = c$
   d) $\frac{x}{x+y} + \log x = c$

4. General solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$ is:
   a) $\log(xy) = c$
   b) $\log y = cx$
   c) $\log\left(\frac{y}{x}\right) = cx$
   d) $\log x = cy$

5. Solution of the differential equation $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x^2\cos x$ is :
   a) $e^{\frac{y}{x}} - \sin x = c$
   b) $e^{\frac{y}{x}} + \sin x = c$
   c) $e^{-\frac{y}{x}} - \sin x = c$
   d) $e^{\frac{-y}{x}} + \sin x = c$

ANSWERS

1. (b) $\tan^{-1}\frac{y}{x} = \log|x| + c$
2. (d) $x^2 + y^2 = cx^3$
3. (d) $\frac{x}{x+y} + \log x = c$
4. (c) $\log\left(\frac{y}{x}\right) = cx$
5. (a) $e^{\frac{y}{x}} - \sin x = c$

LONG ANWER QUESTIONS

1. Show that the differential equation $2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous. Find the particular solution given that x=0 when y=1.

2. Solve: $\frac{dy}{dx} - 3y\cot x = \sin 2x$;.Find the particular solution when $y = 2\ when\ x = \frac{\pi}{2}$.

3. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x\,dx = 0$, given that y = 1, when x = 0.

4. Solve the Differential Equation $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1}x$.

5. In a bank Principal increases at the rate of r % per year. Find the value of r if Rs. 100 double itself in 10 years (given log 2 =0.6931).

6. Solve the differential equation $xdy - y\,dx = \sqrt{x^2 + y^2}\,dx$.

7. Find the particular solution of the Differential Equation $(\tan^{-1} y - x)\,dy = (1 + y^2)dx$, given that x =0, y =0.

8. Find the general solution of the differential equation
$$\frac{dy}{dx} - y = \cos x$$

9. Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1,\ x \neq 0$

10. Solve the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0,\ x \neq 0$

11. Find a particular solution of the differential equation

$$\frac{dy}{dx} - 3y\,Cotx = \text{Sin}\,2x, \text{ given y = 2 when x = } \frac{\pi}{2}$$

12. Find a particular solution of the differential equation

$$\frac{dy}{dx} + y\,Cotx = 2x + x^2 Cotx, x \neq 0, \text{ given y = 0 when } x = \frac{\pi}{2}$$

13. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that

y = 0 when x = 0

14. Find a particular solution of the differential equation

$$\left[x\text{Sin}^2\left(y/x\right) - y\right]dx + xdy = 0 \text{ , given } y = \frac{\pi}{4} \text{ when x = 1}$$

15. Find a particular solution of the differential equation

$$2xy + y^2 - 2x^2\frac{dy}{dx} = 0 \text{ , given y = 2 when x = 1}$$

ANSWERS:

ANSWERS OF MULTIPLE CHOICE QUESTIONS :

1) D 2) D 3) B 4) B 5) A

ANSWERS OF SHORT ANSWER QUESTIONS

1) $2^{-x} - 2^{-y} = k$

2) $\frac{e^6 + 9}{2}$

3) y(x^2- 1) = $\frac{1}{2} log\left(\left|\frac{x-1}{x+1}\right|\right) + k$

4) y = c.$e^{x\,-\,x^2}$

5) (a + m) y = $e^{mx} + ce^{-ax}$

6) (x - c)$e^{x\,+y} + 1 = 0$

7) $y = kxe^{\frac{-x^2}{2}}$

8) y = tan $\left(x+\frac{x^2}{2}\right)$

9) x = y (y^2+c)

10) $\frac{1}{3}$

11) $y=\frac{4x^3}{3(1+x^2)}$

12) $tan^{-1}\left(\frac{y}{x}\right)=log|x|+c$

13) $tan^{-1}\left(\frac{x}{y}\right)+log\ y=c$

14) x + y = ke^{x-y}

15) $x^2\ (y+3)^3=e^{y+2}$

ANSWERS OF LONG ANSWER QUESTIONS:

1. $2e^{\frac{x}{y}}+\log y = 2$

2. y(cosec ^{3} x) = -2 cosecx +4

3. tan $^{-1}$ y = - tan^{-1} e^x + $\frac{\pi}{2}$

4. $e^{tan^{-1}x}(y+1)$ = tan^{-1}x $e^{tan^{-1}x}$ +c

5. r =10 log 2 = 6.931%

6. $y+\sqrt{x^2+y^2}$ = c x^2

7.

8. $x\,e^{tan^{-1}y}=(tan^{-1}y-1)e^{tan^{-1}y}+1$

$y=\frac{1}{2}(\sin x-\cos x)+ce^{x}+c$

9. $y\bullet e^{2\sqrt{x}}=2\sqrt{x}+c$

10. $y=cotx+\frac{1}{x}+\frac{c}{x\,sinx}$

11. $y = 4\sin^3 x - 2\sin^2 x$

12. $y\sin x = x^2 \sin x - \frac{\pi^2}{4}$

13. $-3e^{-4y} = 4e^{3x} - 7$

14. $\log|x| = \cot\left(\frac{y}{x}\right) - 1$

15. $\log|x| = \frac{-2x}{y} + 2$

CHAPTER -10- VECTOR ALGEBRA

SUMMARY

- DEFINITION OF Vectors
- Definition of Scalars
- Position Vector
- Dc's and Dr's
- Types of vectors
    - Zero Vector
    - Unit Vector
    - Unit vector in the direction of vectors
    - Collinear Vectors
    - Coinitial vectors
    - Equal Vectors
    - Negative of a Vector
    - Unit Vectors along the coordinate axes.
- Addition of Vectors- Triangle law of addition / parallelogram law of addition
- Multiplication of a Vector by a Scalar
- Vector joining two points
- Section formula and mid point formula
- Dot product of vectors
- Properties of dot product of vectors
- Projection of vectors on a line
- Perpendicular vectors
- Finding the angle between the two vectors
- Expressing dot product in rectangular coordinates.
- Cross product of vectors
- Properties of Cross product of vectors
- Expressing cross product in rectangular coordinates.
- Unit vector perpendicular to two given vectors.
- Angle between two vectors
- Area of parallelogram when adjacent sides are given.
- Area of parallelogram when diagonals are given.
- Area of a triangle
- Area of a rectangle ABCD, when position vectors of A,B,C,D are given

MIND MAPPING

- Position vector of a point P(x, y, z) is given as $\overline{O\ P}(=\overline{r})$= x $\hat{\imath}$ +y $\hat{\jmath}$ +z $\hat{k}$and its magnitude by $\sqrt{x^2+y^2+z^2}$.
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l=\frac{a}{r}$, $m=\frac{b}{r}$, $n=\frac{c}{r}$
- $l^2+m^2+n^2=1$ but $a^2+b^2+c^2\neq 1$ in general
- The vector sum of the three sides of a triangle taken in order is $\vec{0}$.
- The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple |λ|, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\vec{a}$, $\hat{a}=\frac{\vec{a}}{|a|}$ gives the unit vector in the direction of $\vec{a}$.
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are respectively, in the ratio m : n

(i) internally, is given by $\frac{m\vec{b}+n\vec{a}}{m+n}$

(ii) externally, is given by $\frac{m\vec{b}-n\vec{a}}{m-n}$.

- $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ are parallel or collinear iff $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

Dot product	Cross product
$\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}| \cos\theta$	$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}| \sin\theta\,\hat{n}$ where $\hat{n}$ is perpendicular to both $\vec{a}, b$.
$\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (commutative property)	$\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ (not commutative)
If $\vec{a}\perp\vec{b}$ $then\ \vec{a}.\vec{b} = 0$	If $\vec{a}\parallel\vec{b}$, $then\ \vec{a}\times\vec{b} = 0$
$\vec{a}.\vec{a} = |\vec{a}|^2$ $(\vec{a}.\vec{b})^2 = (\vec{a})^2 + (\vec{b})^2 + 2\vec{a}.\vec{b}$	$\vec{a}\times\vec{a} = 0$
If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ Then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$	If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ Then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
Geometrical meaning projection of $\vec{a}\ on\ \vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$	Geometrical meaning $\vec{a}\times\vec{b} = vector\ area\ of\ a$ parallelogram with $\vec{a}, \vec{b}$ represent the adjacent sides.
$\hat{i}\,.\,\hat{i} = \hat{j}\,.\,\hat{j} = \hat{k}\,.\,\hat{k} = 1$	$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$
$\hat{i}\,.\,\hat{j} = \hat{j}\,.\,\hat{k} = \hat{i}\,.\,\hat{k} = 0$	$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

MCQ Questions

1. **If points A ($60\,\hat{\imath} + 3\,\hat{\jmath}$), ($40\,\hat{\imath} – 8\,\hat{\jmath}$) and C ($a\hat{\imath}$- $52\hat{\jmath}$) are collinear, then 'a' is equal to**
   a) **40** **b) -40** **c) 20** **d) -20**
2. **If $\vec{a} + \vec{b} + \vec{c}$ = 0, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$then the angle between If $\vec{a}\,and\,\vec{b}$ is**
   a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) $\frac{5\pi}{3}$ d) $\frac{\pi}{3}$
3. **If $\vec{a}$ = $\hat{\imath}$ + $\hat{\jmath}$ - $\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ +2 $\hat{k}$ and $\vec{c}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ - $\hat{k}$ then a unit vector normal to the vectors ($\vec{a} + \vec{b}$) and ($\vec{b} - \vec{c}$) is**
   a) $\hat{\imath}$ b) $\hat{\jmath}$ c) $\hat{k}$ d) none of these
4. **If $|\vec{a}\ x\ \vec{b}| = 4$, | $\vec{a}\,.\vec{b}$ |=2, then $|\vec{a}|^2\ \ |\vec{b}|^2$ is**
   a) 6 b) 2 c) 20 d) 8
5. **The value of $\hat{\imath}$. ($\hat{\jmath}$ x $\hat{k}$) + $\hat{\jmath}$. ($\hat{\imath}$ x $\hat{k}$) + $\hat{k}$. ($\hat{\imath}$ x $\hat{\jmath}$) is**
   a) 0 b) -1 c) 1 d) 3

Short Answer Questions

1. **Find the unit vector in the direction of the sum of the vectors $\vec{a}$ = 2$\hat{\imath}$ +2 $\hat{\jmath}$ -5 $\hat{k}$, $\vec{b}$ = 2$\hat{\imath}$ + $\hat{\jmath}$ +3 $\hat{k}$.**
2. **Show that the points A(-2$\hat{\imath}$ +3 $\hat{\jmath}$ +5 $\hat{k}$), B ($\hat{\imath}$ +2 $\hat{\jmath}$ +3 $\hat{k}$), C (7$\hat{\imath}$ - $\hat{k}$) are collinear.**
3. **If $\vec{a}$ = 2 $\hat{\imath}$ + 2 $\hat{\jmath}$ +3 $\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ + $\hat{k}$, $\vec{c}$ = 3 $\hat{\imath}$ + $\hat{\jmath}$ are such that $\vec{a}$ +λ $\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.**
4. **Find the area of the ||gm whose adjacent sides are represented by the vectors $\vec{a}$ = 3 $\hat{\imath}$ + $\hat{\jmath}$ -2 $\hat{k}$, $\vec{b}$ = $\hat{\imath}$ -3 $\hat{\jmath}$ +4 $\hat{k}$**
5. **Find a vector of magnitude 3√2 units which makes an angle of π/4, π/2 with y and z-axes, respectively.**
6. **If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a}\ x\ \vec{b}| = 8$, find $\vec{a}\,.\vec{b}$**
7. **If $\vec{a}$ = 2 $\hat{\imath}$ -3 $\hat{\jmath}$ +$\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + $\hat{k}$, $\vec{c}$ = 2 $\hat{\jmath}$ -$\hat{k}$ are three vectors, find the area of the parallelogram having diagonals ($\vec{a} + \vec{b}$) and ($\vec{b} + \vec{c}$)**
8. **If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a}\ x\ \vec{b} = 3\,\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\vec{a}\ and\ \vec{b}$**
9. **If $\vec{p}$ and $\vec{q}$ are the unit vectors forming an angle of 30^0 , find the area of the parallelogram having $\vec{a}$ = $\vec{p} + 2\,\vec{q}$ $and\ \vec{b}$ = $2\vec{p} + \vec{q}$ as its diagonals.**
10. **$\vec{a}$ is a unit vector and ($\vec{x} - \vec{a}$) ($\vec{x} + \vec{a}$) = 8, then find $|\vec{x}|$**

Long Answer Questions

1. **If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c}$ = 0,then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c}$ + $\vec{c}\,.\vec{a}$**
2. **The scalar product of the vector $\hat{\imath}$ + $\hat{\jmath}$ + $\hat{k}$ with a unit vector along the sum of vectors 2 $\hat{\imath}$ + 4 $\hat{\jmath}$ -5 $\hat{k}$ and λ$\hat{\imath}$ + 2 $\hat{\jmath}$ +3 $\hat{k}$ is equal to one. Find the value of λ.**
3. **Let $\vec{a}, \vec{b}\ and\ \vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two find | $\vec{a} + \vec{b} + \vec{c}$ |**

4. . If with reference to the righthanded system of mutually perpendicular unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$, $\vec{\alpha}$ = $3\hat{\imath}$ - $\hat{\jmath}$ and $\vec{\beta}$ = 2 $\hat{\imath}$ + $\hat{\jmath}$ -3 $\hat{k}$ $t$hen express $\vec{\beta}$ in the form $\vec{\beta} = \overrightarrow{\beta_1}$ + $\overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$
5. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}$ + $\vec{b}$ + $\vec{c}$ = 0 and | $\vec{a}| = 3$, | $\vec{b}| = 5$, | $\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$
6. If $\vec{a}$ x $\vec{b}$ = $\vec{c}$ x $\vec{d}$ and $\vec{a}$ x $\vec{c}$ = $\vec{b}$ x $\vec{d}$, then show that ($\vec{a}$ - $\vec{d}$) is parallel to ($\vec{b}$ - $\vec{c}$) , given that $\vec{a}$ ≠ $\vec{d}$ and $\vec{b}$ ≠ $\vec{c}$.
7. Find the area of the parallelogram whose diagonals are $\vec{a}$ and $\vec{b}$. Also find the area of the parallelogram whose diagonals are 2 $\hat{\imath}$ - $\hat{\jmath}$ + $\hat{k}$ and $\hat{\imath}$ + 3 $\hat{\jmath}$ - $\hat{k}$.
8. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}\right]$ gives the vector area of triangle. Hence deduce the condition that the three points $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.
9. Given that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a triangle such that $\vec{a}$ = $\vec{b}$ + $\vec{c}$. Find p,q,r,s such that area of triangle is 5 $\sqrt{6}$ where $\vec{a}$ = p$\hat{\imath}$ +q $\hat{\jmath}$ +r $\hat{k}$, $\vec{b}$ = s $\hat{\imath}$ + 3 $\hat{\jmath}$ + 4$\hat{k}$ and $\vec{c}$ = 3 $\hat{\imath}$ + $\hat{\jmath}$ -2$\hat{k}$.
10. If $\vec{a}$ = $\hat{\imath}$ + $\hat{\jmath}$ + $\hat{k}$ and $\vec{b}$ = $\hat{\jmath}$ -$\hat{k}$, then find a vector $\vec{c}$ such that $\vec{a}$ x $\vec{c}$ = $\vec{b}$ and $\vec{a}$. $\vec{c}$ = 3.

CASE STUDY QUESTIONS

CASE STUDY - I

Ginni purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A (1, 1, 1), B (2, 1, 3), C=(3, 2, 2) and D = (3,3,4).
Based on the above information, answer the following questions

1. Find the position vector of AB.
   a. -$\hat{\imath}$-2 $\hat{k}$
   b. 2$\hat{\imath}$+ $\hat{k}$
   c. $\hat{\imath}$+2 $\hat{k}$
   d. -2$\hat{\imath}$- $\hat{k}$

2. Find the position vector of AC.
   a. 2$\hat{\imath}$- $\hat{\jmath}$- $\hat{k}$
   b. 2$\hat{\imath}$+ $\hat{\jmath}$+ $\hat{k}$
   c. -2$\hat{\imath}$- $\hat{\jmath}$+ $\hat{k}$
   d. î+2 $\hat{\jmath}$+ $\hat{k}$

3. Find the position vector of AD
   a. 2$\hat{\imath}$-2 $\hat{\jmath}$-3 $\hat{k}$
   b. $\hat{\imath}$+ $\hat{\jmath}$-3 $\hat{k}$
   c. 3$\hat{\imath}$+2 $\hat{\jmath}$+2 $\hat{k}$
   d. 2$\hat{\imath}$+2 $\hat{\jmath}$+3 $\hat{k}$

4. Area of ΔABC=

a. $\frac{\sqrt{11}}{2}$ sq. units
b. $\frac{\sqrt{14}}{2}$ sq. units
c. $\frac{\sqrt{13}}{2}$ sq. units
d. $\frac{\sqrt{17}}{2}$ sq. units

CASE STUDY - II

Team A,B,C went for playing a tug of war game. Teams A, B, C, have attached a rope to a mental ring and its trying to pull the ring into their own area(learn areas shown below).

Team A pulls with force F1=4$\hat{\imath}$+0ĵ KN

Team B →F2= 2î+4ĵ KN

Team C →F3=-3$\hat{\imath}$–3ĵ KN

Based on the above information, answer the following.

1. **Which team will win the game?**
   - a. **Team B**
   - b. **Team A**
   - c. **Team C**
   - d. **No one**
2. **What is the magnitude of the teams combined force?**
   - a. **7 KN**
   - b. **1.4 KN**
   - c. **1.5 KN**
   - d. **2 KN**
3. **In what direction is the ring getting pulled?**
   - a. **2.0 radian**
   - b. **2.5 radian**
   - c. **2.4 radian**
   - d. **3 radian**
4. **What is the magnitude of the forces of Team B?**
   - a. **2√5 KN**
   - b. **6 KN**
   - c. **2 KN**
   - d. **√6KN**
5. **How many KN force is applied by Team A?**
   - a. **5 KN**

b. 4 KN
c. 2 KN
d. 16 KN

CASE STUDY – III

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ *be three* non zero vectors.

1. **If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then**
   - a. $\vec{a} \perp \vec{b}$
   - b. $\vec{a} || \vec{b}$
   - c. $\vec{a}=\vec{b}$
   - d. None of these
2. **If $\vec{a}=\hat{\imath}-2\hat{\jmath}$,$\vec{b}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$ then evaluate $(2\vec{a}+\vec{b})\cdot[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$**
   - a. 0
   - b. 4
   - c. 3
   - d. 2
3. **If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then**

   $|\vec{a}-\vec{b}|$

   - a. $\sin\frac{\theta}{2}$
   - b. $2\sin\frac{\theta}{2}$
   - c. $2\cos\frac{\theta}{2}$
   - d. $\cos\frac{\theta}{2}$

4. Let $\vec{a}\,,\vec{b}\ and\ \vec{c}$ be unit vectors such that $\vec{a}\,.\vec{b} = \ \vec{a}\,.\vec{c} = 0$ and angle between $\vec{b}\ and\ \vec{c}\ is\frac{\pi}{6}\ then\ \vec{a} =$

   a. $2\,(\vec{b}\times\vec{c})$

   b. $-2\,(\vec{b}\times\vec{c})$

   c. $\pm 2\,(\vec{b}\times\vec{c})$

   d. $2\,(\vec{b}\pm\vec{c})$

5. The area of the parallelogram If $\vec{a}= \hat{i}-2\hat{j}$,$\vec{b}=2\hat{i}+\hat{j}+3\hat{k}$ as diagonals is

   a. 70

   b. 35

   c. √70/2

   d. √70

ANSWERS

MCQ QUESTIONS

1. b 2. d 3. a 4. c 5. c

SHORT ANSWER QUESTIONS

1) $\frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{k}$ 3) 8 4) $10\sqrt{3}$ 5) $\pm 3\hat{\imath} + 3\hat{\jmath}$ 6) 6

7) $\frac{\sqrt{21}}{2}$ 8) $\frac{\pi}{6}$ 9) $\frac{3}{4}$ 10) 3

LONG ANSWER QUESTIONS

1) $\frac{-3}{2}$ 2) 1 3) $5\sqrt{2}$ 4) λ=1/2 , $\overrightarrow{\beta_1} = \frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}$ $\overrightarrow{\beta_2} = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$

5) $\frac{\pi}{3}$ 7) $\frac{1}{2}\sqrt{62}$ 9) p = -8 , q=4, r=2 , s= -11,5 10) $\frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$

CASE STUDY – I ANSWERS

1. (c)   2. (b)   3. (d)   4. (b)   5. (a)

CASE STUDY – II ANSWERS

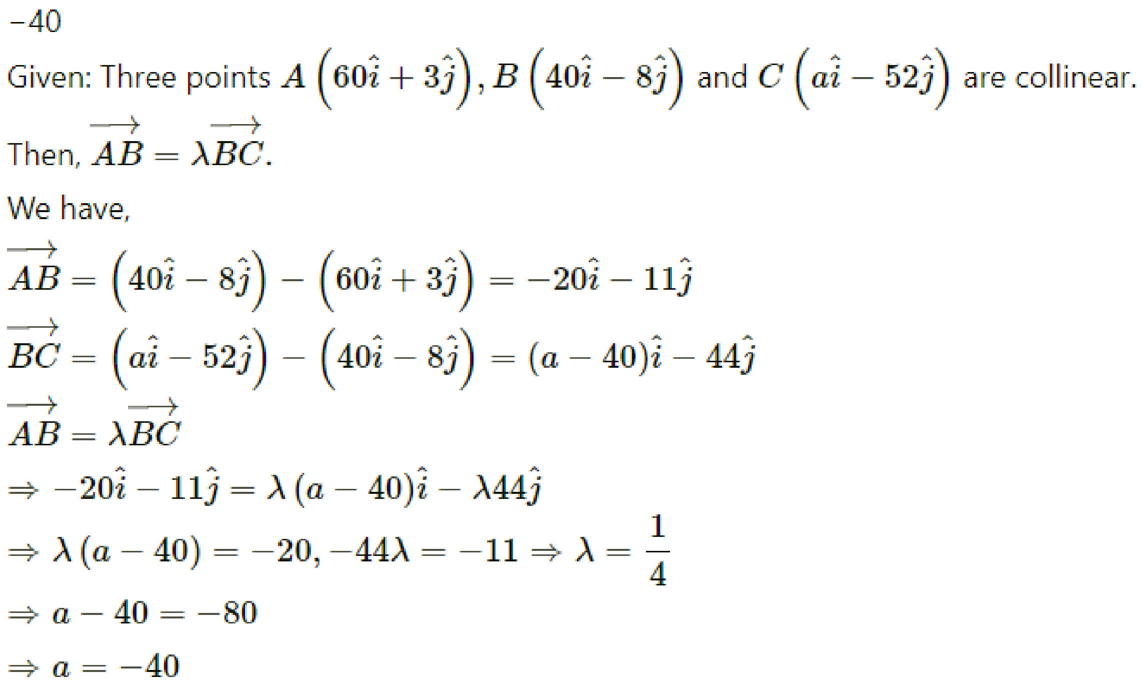
1. a. Team B   2. b. 1.4 KN   3. c. 2.4 KN   4. a. 2v5 KN   5. b. 4 KN

CASE STUDY – III ANSWERS

1. a   2. A   3. B   4. C   5. C

MCQ Questions - solutions

1.



−40

Given: Three points $A\left(60\hat{i}+3\hat{j}\right), B\left(40\hat{i}-8\hat{j}\right)$ and $C\left(a\hat{i}-52\hat{j}\right)$ are collinear.

Then, $\overrightarrow{AB}=\lambda\overrightarrow{BC}$.

We have,

$$\overrightarrow{AB}=\left(40\hat{i}-8\hat{j}\right)-\left(60\hat{i}+3\hat{j}\right)=-20\hat{i}-11\hat{j}$$

$$\overrightarrow{BC}=\left(a\hat{i}-52\hat{j}\right)-\left(40\hat{i}-8\hat{j}\right)=(a-40)\hat{i}-44\hat{j}$$

$$\overrightarrow{AB}=\lambda\overrightarrow{BC}$$

$$\Rightarrow -20\hat{i}-11\hat{j}=\lambda\,(a-40)\hat{i}-\lambda 44\hat{j}$$

$$\Rightarrow \lambda\,(a-40)=-20, -44\lambda=-11\Rightarrow\lambda=\frac{1}{4}$$

$$\Rightarrow a-40=-80$$

$$\Rightarrow a=-40$$

2.

Given , $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5$ and $\left|\vec{c}\right| = 7 \ldots (i)$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Given that

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$$

$$\Rightarrow \left|\vec{a} + \vec{b}\right| = \left|-\vec{c}\right|^2$$

$$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} = \left|\vec{c}\right|^2$$

$$\Rightarrow 2\vec{a}.\vec{b} = \left|\vec{c}\right|^2 - \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2$$

$$\Rightarrow 2\vec{a}.\vec{b} = 7^2 - 3^2 - 5^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [\text{ Using } (i)]$$

$$\Rightarrow 2\vec{a}.\vec{b} = 15$$

$$\Rightarrow 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 15$$

$$\Rightarrow 2(3)(5)\cos\theta = 15 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [\text{ Using } (i)]$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

3.

$$\vec{a}+\vec{b}=0\hat{i}+3\hat{j}+\hat{k}$$

$$\vec{b}-\vec{c}=0\hat{i}-0\hat{j}+3\hat{k}$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 3 & 1\\ 0 & 0 & 3\end{vmatrix}$$

$$=9\hat{i}$$

$$\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)\right|=9\left|\hat{i}\right|$$

$$=9\left(1\right)$$

$$=9$$

Unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}=\dfrac{\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)}{\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)\right|}$

$$=\frac{9\hat{i}}{9}$$

$$=\hat{i}$$

4.

We know

$$\left(\vec{a}.\vec{b}\right)^2+\left|\vec{a}\times\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2\ldots(1)$$

$$\left|\vec{a}.\vec{b}\right|=2(\text{ Given })$$

$$\Rightarrow\left|\vec{a}.\vec{b}\right|^2=\left(\vec{a}.\vec{b}\right)^2$$

From (1), we get

$$(2)^2+(4)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

$$\Rightarrow\left|\vec{a}\right|^2\left|\vec{b}\right|^2=20$$

5.

$$\begin{aligned}&\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)\\&=\hat{i}.\hat{i}+\hat{j}.\left(-\hat{j}\right)+\hat{k}.\hat{k}\\&=\left|\hat{i}\right|^2-\left|\hat{j}\right|^2+\left|\hat{k}\right|^2\\&=1-1+1\\&=1\end{aligned}$$

Short Answer Type – Solutions

1. **Ans: Let** $\vec{c}=\vec{a}+\vec{b}$

$$=\left(2\hat{i}+2\hat{j}-5\hat{k}\right)+\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

$$=4\hat{i}+3\hat{j}-2\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{16+9+4}$$

$$=\sqrt{29}$$

The required unit vector is

$$\hat{c}=\frac{\vec{c}}{\left|\vec{c}\right|}$$

$$=\frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{29}}$$

$$=\frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$$

2. Ans: $\overrightarrow{AB}=3\hat{i}-\hat{j}-2\hat{k}$

$\overrightarrow{BC}=6\hat{i}-2\hat{j}-4\hat{k}$

$\overrightarrow{CA}=9\hat{i}-3\hat{j}-6\hat{k}$

$\left|\overrightarrow{AB}\right|=\sqrt{14},\overrightarrow{BC}=2\sqrt{14}$

$and\left|\overrightarrow{AC}\right|=3\sqrt{14}$

$\left|\overrightarrow{AC}\right|=\left|\overrightarrow{AB}\right|+\left|\overrightarrow{BC}\right|$

Hence points A, B, C are collinear.

3. Ans: $\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$

$=\left(2-\lambda\right)\hat{i}+\left(2+2\lambda\right)\hat{j}+\left(3+\lambda\right)\hat{k}$

$\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0\left[\because\vec{a}+\lambda\vec{b}\perp\vec{c}\right.$

$\left[\left(2-\lambda\right)\hat{i}+\left(2+2\lambda\right)\hat{j}+\left(3+\lambda\right)\hat{k}\right].\left(3\hat{i}+\hat{j}\right)=0$

$3\left(2-\lambda\right)+\left(2+2\lambda\right)=0$

$-\lambda=-8$

$\lambda=8$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}$$

4. Ans:

$=-2\hat{i}-14\hat{j}-10\hat{k}$

$req.area=\left|\vec{a}\times\vec{b}\right|$

$=\sqrt{\left(-2\right)^2+\left(-14\right)^2+\left(-10\right)^2}=10\sqrt{3}$

5. Solution:

From the give,

m = cos π/4 = 1/√2

n = cos π/2 = 0

Therefore, $l^2 + m^2 + n^2 = 1$

l^2 + (½) + 0 = 1

l^2 = 1 – ½

l = ±1/√2

Hence, the required vector is:

$$\vec{r} = 3\sqrt{2}\left(l\hat{\imath} + m\hat{\jmath} + n\hat{k}\right)$$

$$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath} + 0\hat{k}\right)$$

$$\vec{r} = \pm 3\hat{\imath} + 3\hat{\jmath}$$

6.

We know

$$\left(\vec{a}.\vec{b}\right)^2 + \left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 + 8^2 = 2^2 \times 5^2 (\because \left|\vec{a} \times \vec{b}\right| = 8, \left|\vec{a}\right| = 2 \text{ and } \left|\vec{b}\right| = 5)$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 + 64 = 100$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 = 36$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right) = 6$$

7.

It is given that $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{b} = -\hat{i} + \hat{k}, \vec{c} = 2\hat{j} - \hat{k}$

$$\therefore \vec{a} + \vec{b} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right) + \left(-\hat{i} + \hat{k}\right) = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} + \vec{c} = \left(-\hat{i} + \hat{k}\right) + \left(2\hat{j} - \hat{k}\right) = -\hat{i} + 2\hat{j}$$

We know that the area of parallelogram is $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$, where $\vec{d_1}$ and $\vec{d_2}$ are the diagonal vectors.

Now,

$$\left(\vec{a} + \vec{b}\right) \times \left(\vec{b} + \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i} - 2\hat{j} - \hat{k}$$

$\therefore$ Area of the parallelogram having diagonals $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{b} + \vec{c}\right)$

$$= \frac{1}{2}\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)\right|$$
$$= \frac{1}{2}\left|-4\hat{i}-2\hat{j}-\hat{k}\right|$$
$$= \frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2}$$
$$= \frac{\sqrt{21}}{2} \text{ square units}$$

Thus, the required area of the parallelogram is $\frac{\sqrt{21}}{2}$ square units.

8.

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$$\vec{a}\times\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}(\text{ Given })$$
$$\Rightarrow \left|\vec{a}\times\vec{b}\right| = \sqrt{9+4+36}$$
$$= 7$$

We know

$$\left|\vec{a}\times\vec{b}\right| = |\vec{a}|\left|\vec{b}\right|\sin\theta$$
$$\Rightarrow 7 = (2)(7)\sin\theta$$
$$\Rightarrow \sin\theta = \frac{1}{2}$$
$$\Rightarrow \theta = \frac{\pi}{6}$$

9.

$$\vec{a} = \hat{p} + 2\ \widehat{q}$$
$$\vec{b} = 2\ \widehat{p}\ + \hat{q}$$
$$\vec{a} \times \vec{b} = (\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})$$
$$= 2\vec{p} \times \vec{p} + \vec{p} \times \vec{q} + 4\vec{q} \times \vec{p} + 2\vec{q} \times \vec{q}$$
$$= 2\,(0) + \vec{p} \times \vec{q} - 4\vec{p} \times \vec{q} + 2\,(0)$$
$$= -3\vec{p} \times \vec{q}$$

Area of the parallelogram $= \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$

$$= \frac{1}{2}\left|-3\,(\vec{p} \times \vec{q})\right|$$
$$= \frac{3}{2}\left|\vec{p}\right|\,\left|\vec{q}\right| \sin 30^{o}$$
$$= \frac{3}{2}(1)\,(1)\left(\frac{1}{2}\right)(\because \vec{p} \text{ and } \vec{q} \text{ are unit vectors })$$
$$= \frac{3}{4} \text{ sq. units}$$

10

Ans: $\left|\vec{a}\right| = 1$

$$\left(\vec{x} - \vec{a}\right).\left(\vec{x} + \vec{a}\right) = 8$$
$$\left|\vec{x}\right|^2 - 1 = 8$$
$$\left|\vec{x}\right|^2 = 9$$
$$\left|\vec{x}\right| = 3$$

Long Answer Questions – solutions

1. **Ans:** $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 1,$

$$\vec{a} + \vec{b} + \vec{c} = 0 \quad (Given)$$
$$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right)$$
$$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$
$$\left(\vec{a}\right)^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$
$$1 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$

$$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1-------(i)$$

similiorly

$$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1-------(ii)$$

again

$$\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1-------(iii)$$

adding(i),(ii)and(iii)

$$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-3 \qquad \left[\vec{a}.\vec{b}=\vec{b}.\vec{a}\right.$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$$

2. **Ans:** $\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}$

$$\vec{b}=\lambda\hat{i}+\hat{j}+3\hat{k}$$

$$\vec{a}+\vec{b}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$$

Unit vector along

$$\vec{a}+\vec{b}=\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+\ (6)^2\ +(-2)^2}}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+\ 40}}$$

$$ATQ\ \vec{c}.\left(\vec{a}+\vec{b}\right)=1$$

$$\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{(2+\lambda)^2+40}\right)=1$$

$$\frac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}}=1$$

$$2+\lambda+4=\sqrt{(2+\lambda)^2+40}$$

sq.both site

$$\lambda^2+36+12\lambda=(2+\lambda)^2+40$$

$$\lambda=1$$

3. **Ans:** $\vec{a}.\left(\vec{b}+\vec{c}\right)=0,\ \vec{b}.\left(\vec{c}+\vec{a}\right)=0\ \vec{c}.\left(\vec{a}+\vec{b}\right)=0,(Given)$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$$

$$=\vec{a}.\vec{a}+\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}.\left(\vec{a}+\vec{c}\right)+\vec{c}.\vec{c}+\left(\vec{a}+\vec{b}\right)$$

$$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2$$

$$= 9 + 16 + 25$$

$$= 50$$

$$\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{50}$$

$$= 5\sqrt{2}$$

4. **Ans:** $Let\ \vec{\beta}_1 = \lambda\vec{\alpha}$ $\left[\because \vec{\beta}_1 \parallel to \vec{\alpha}\right.$

$$\vec{\beta}_1 = \lambda\left(3\hat{i} - \hat{j}\right)$$

$$= 3\lambda\hat{i} - \lambda\hat{j}$$

$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$

$$= \left(2\hat{i} + \hat{j} - 3\hat{k}\right) - \left(3\lambda\hat{i} - \lambda\hat{j}\right)$$

$$= (2 - 3\lambda)\hat{i} + (1 + \lambda)\hat{j} - 3\hat{k}$$

$$\vec{\alpha}.\vec{\beta}_2 = 0 \qquad \left[\because \vec{\beta}_2 \perp \vec{\alpha}\right.$$

$$3(2 - 3\lambda) - (1 + \lambda) = 0$$

$$\lambda = \frac{1}{2}$$

$$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$$

5. **Ans:** $\vec{a} + \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\left(\vec{a} + \vec{b}\right).\left(-\vec{c}\right) = -\vec{c}.\left(-\vec{c}\right)$$

$$\left(\vec{a} + \vec{b}\right).\left(\vec{a} + \vec{b}\right) = \vec{c}.\vec{c}$$

$$\left|\vec{a}\right|^2 + 2\vec{a}\vec{b} + \left|\vec{b}\right|^2 = \left|\vec{c}\right|^2$$

$$\vec{a}.\vec{b} = \frac{49 - 9 - 25}{2} = \frac{15}{2}$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$= \frac{1}{2}$$

$$\theta = 60$$

6.

Sol. Given, $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow$ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}$

$\Rightarrow$ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}=\vec{0}$

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d}=\vec{0}$ [By left and right distributive law]

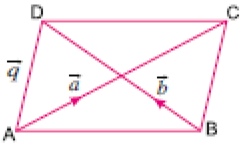
$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=\vec{0}$ [$\because$ $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$]

$\Rightarrow$ $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$ [By right distributive law]

$\Rightarrow$ $(\vec{a}-\vec{d})\ ||\ (\vec{b}-\vec{c})$

7

Sol. Let $ABCD$ be a parallelogram such that



$$\overrightarrow{AB}=\vec{p},\ \overrightarrow{AD}=\vec{q}\ \Rightarrow\ \overrightarrow{BC}=\vec{q}$$

By triangle law of addition, we get

$$\overrightarrow{AC}=\vec{p}+\vec{q}=\vec{a} \qquad \text{[say]} \qquad ...(i)$$

Similarly, $$\overrightarrow{BD}=-\vec{p}+\vec{q}=\vec{b} \qquad \text{[say]} \qquad ...(ii)$$

On adding equation (*i*) and (*ii*), we get

$$\vec{a}+\vec{b}=2\vec{q}\ \Rightarrow\ \vec{q}=\frac{1}{2}(\vec{a}+\vec{b})$$

On subtracting equation (*ii*) from equation (*i*), we get

$$\vec{a}-\vec{b}=2\vec{p}\ \Rightarrow\ \vec{p}=\frac{1}{2}(\vec{a}+\vec{b})$$

Now, $$\vec{p}\times\vec{q}=\frac{1}{4}(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=\frac{1}{4}(\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b})$$

$$=\frac{1}{4}[\vec{a}\times\vec{b}+\vec{a}\times\vec{b}]=\frac{1}{2}(\vec{a}\times\vec{b})$$

So, area of a parallelogram $ABCD=\left|\vec{p}\times\vec{q}\right|=\frac{1}{2}|\vec{a}\times\vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

$$=\frac{1}{2}|(2\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+3\hat{j}-\hat{k})|$$

$$=\frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}\right|$$

$$=\frac{1}{2}|[\hat{i}(1-3)-\hat{j}(-2-1)+\hat{k}(6+1)]|$$

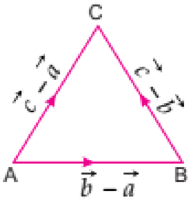
$$=\frac{1}{2}|-2\hat{i}+3\hat{j}+7\hat{k}|$$

$$=\frac{1}{2}\sqrt{4+9+49}$$

$$=\frac{1}{2}\sqrt{62}\text{ sq. units}$$

8

Sol. Since, $\vec{a}$ $\vec{b}$ and $\vec{c}$ are the vertices of a ΔABC as shown.



$\therefore$ Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}$ and $\overrightarrow{AC}=\vec{c}-\vec{a}$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]$

$$= \frac{1}{2}\left|(\vec{b} \times \vec{c}) - (\vec{b} \times \vec{a}) - (\vec{a} \times \vec{c}) + (\vec{a} \times \vec{a})\right|$$

$$= \frac{1}{2}\left|(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{a}) + \vec{0}\right|$$

$$= \frac{1}{2}\left|(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{a})\right| \qquad ...(i)$$

For three points to be collinear, area of the ΔABC should be equal to zero.

$\Rightarrow \qquad \frac{1}{2}[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}] = 0$

$\Rightarrow \qquad \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b} = 0 \qquad ...(ii)$

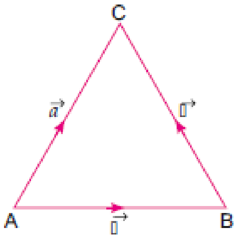
This is the required condition for collinearity of three points $\vec{a}, \vec{b}$ and $\vec{c}$.

Let $\hat{n}$ be the unit vector normal to the plane of the ΔABC.

$\therefore \qquad \hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$

9

Sol. Given, $\vec{a} = \vec{b} + \vec{c}$



$\Rightarrow \qquad p\hat{i} + q\hat{j} + r\hat{k} = (s\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} + \hat{j} - 2\hat{k})$

$\Rightarrow \qquad p\hat{i} + q\hat{j} + r\hat{k} = (s + 3)\,\hat{i} + 4\hat{j} + 2\hat{k}$

Equating the co-efficient of $\hat{i}, \hat{j}, \hat{k}$ from both sides, we get

$\Rightarrow \qquad s + 3 = p; \;\; q = 4 \;$ and $\; r = 2 \qquad ...(i)$

Now, area of triangle = $\frac{1}{2}|\vec{b} \times \vec{c}\,|$

$\Rightarrow \qquad 5\sqrt{6} = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix}\right| = \frac{1}{2}|(-6-4)\hat{i} - (-2s-12)\hat{j} + (s-9)\hat{k}|$

$\Rightarrow \qquad 5\sqrt{6} = \frac{1}{2}\sqrt{10^2 + (2s+12)^2 + (s-9)^2} = \frac{1}{2}\sqrt{100 + 4s^2 + 144 + 48s + s^2 + 81 - 18s}$

$\Rightarrow \qquad 5\sqrt{6} = \frac{1}{2}\sqrt{325 + 5s^2 + 30s}$

Squaring both sides

$\Rightarrow \qquad 150 = \frac{1}{4}(325 + 5s^2 + 30s)$

$\Rightarrow \qquad 600 - 325 = 5s^2 + 30s \qquad \Rightarrow \qquad 5s^2 + 30s - 275=0$

$\Rightarrow \qquad s = \frac{-30 \pm \sqrt{900 + 4\times5\times275}}{10} = \frac{-30 \pm \sqrt{6400}}{10} = \frac{-30 \pm 80}{10}$

$\Rightarrow \qquad s = -11, 5 \qquad ...(ii)$

From (i) and (ii)

$s = -11, 5; \;\; p = -8, 8; \;\; q = 4$ and $r = 2$

10

Sol. Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then,

$$(\vec{a}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ c_1 & c_2 & c_3 \end{vmatrix} = (c_3 - c_2)\hat{i} + (c_1 - c_3)\hat{j} + (c_2 - c_1)\hat{k}$$

$\because \quad (\vec{a}\times\vec{c}) = \vec{b}$

$\Rightarrow \quad (c_3 - c_2)\,\hat{i} + (c_1 - c_3)\,\hat{j} + (c_2 - c_1)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow \quad c_3 - c_2 = 0, c_1 - \ c_3 = 1$ and $c_2 - c_1 = -1$ ...(*i*)

Also, $\quad \vec{a}\,.\,\vec{c} = (\hat{i} + \hat{j} + \hat{k})\,.\,(c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$

$\Rightarrow \quad \vec{a}\,.\,\vec{c} = c_1 + c_2 + c_3$

$\Rightarrow \quad c_1 + c_2 + c_3 = 3$ $[\because \vec{a}\,.\,\vec{c} = 3\ ]$ ...(*ii*)

$\Rightarrow \quad c_1 + c_2 + c_1 - 1 = 3$ $[\because c_1 - c_3 = 1]$ ... (*iii*)

$\Rightarrow \quad 2c_1 + c_2 = 4$

On solving $c_1 - c_2 = 1$ and $2c_1 + c_2 = 4$, we get

$3c_1 = 5 \quad \Rightarrow \quad c_1 = \dfrac{5}{3}$

$\therefore \quad c_2 = (c_1 - 1) = \left(\dfrac{5}{3} - 1\right) = \dfrac{2}{3}$ and $c_3 = c_2 = \dfrac{2}{3}$

Hence, $\vec{c} = \left(\dfrac{5}{3}\hat{i} + \dfrac{2}{3}\hat{j} + \dfrac{2}{3}\hat{k}\right)$.

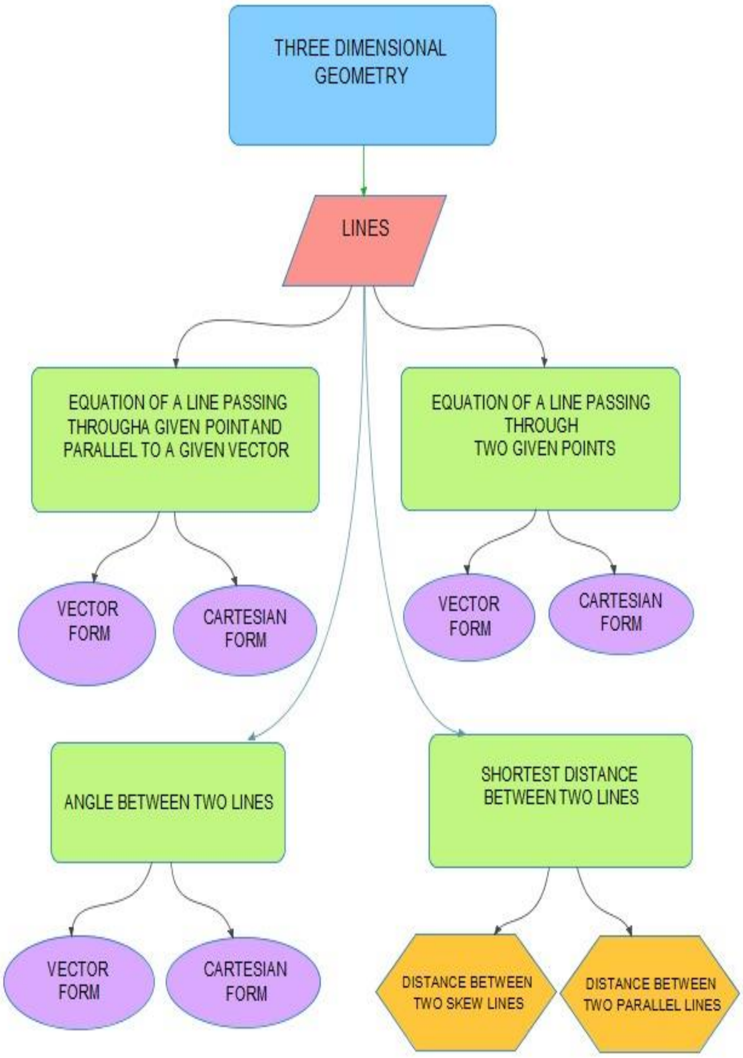
KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

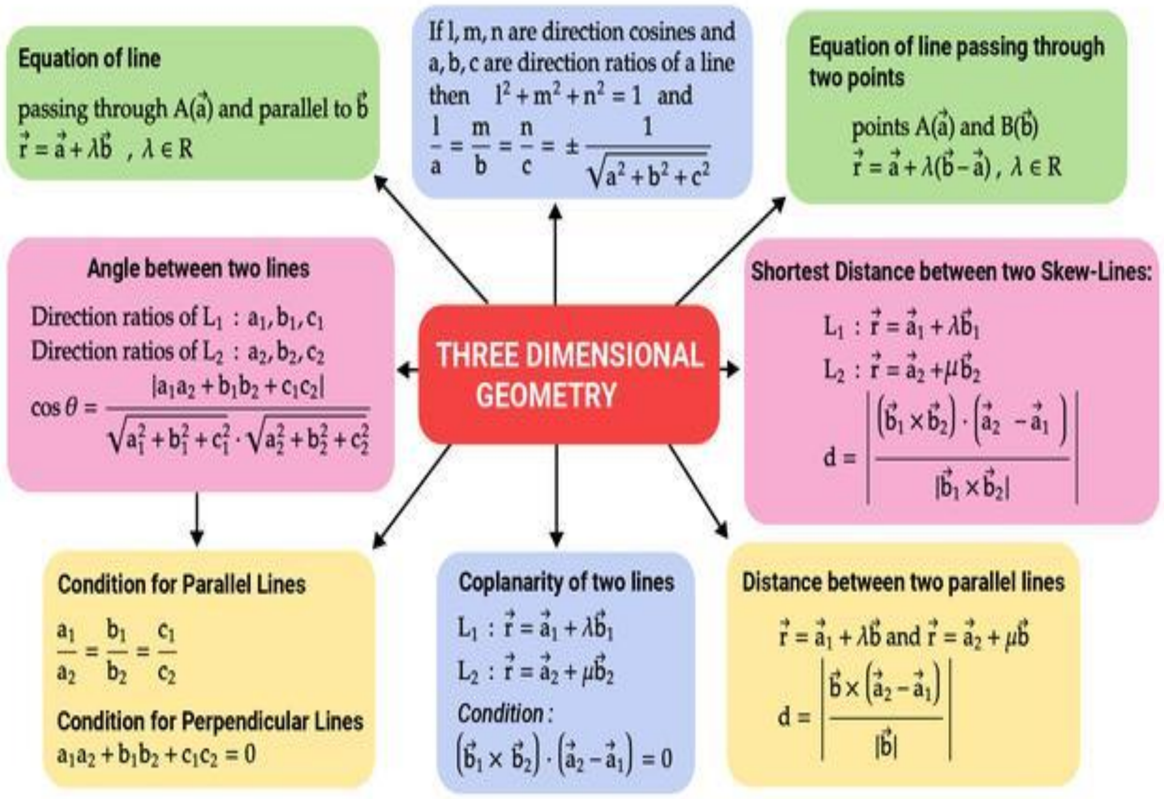
STUDENT SUPPORT MATERIAL 2022-2023

MATHEMATICS

THREE DIMENSIONAL GEOMETRY

CONCEPT MAPPING





Equation of line

passing through $A(\vec{a})$ and parallel to $\vec{b}$

$\vec{r}=\vec{a}+\lambda\vec{b}$, $\lambda\in R$

If l, m, n are direction cosines and a, b, c are direction ratios of a line then $l^2+m^2+n^2=1$ and

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=\pm\frac{1}{\sqrt{a^2+b^2+c^2}}$$

Equation of line passing through two points

points $A(\vec{a})$ and $B(\vec{b})$

$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$, $\lambda\in R$

THREE DIMENSIONAL GEOMETRY

Angle between two lines

Direction ratios of L_1 : a_1, b_1, c_1

Direction ratios of L_2 : a_2, b_2, c_2

$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

Shortest Distance between two Skew-Lines:

L_1 : $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$

L_2 : $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

Condition for Parallel Lines

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Condition for Perpendicular Lines

$a_1a_2+b_1b_2+c_1c_2=0$

Coplanarity of two lines

L_1 : $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$

L_2 : $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

Condition:

$\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)=0$

Distance between two parallel lines

$\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$

$$d=\left|\frac{\vec{b}\times\left(\vec{a}_2-\vec{a}_1\right)}{|\vec{b}|}\right|$$

THINGS TO REMEMBER

Direction Cosines of a line

A directed line *l* passing through origin making angles α, β, γ with x, y and z axes respectively are called direction angles. Cosine of these angles namely cos α, cos β, cos γ are called direction cosines of the directed line *l*.

Direction cosines of a line are denoted by *l,m,n*

l = cos α ,m = cos β , n = cos γ

If *l, m* and *n* are the direction cosine of a line then $l^2+m^2+n^2=1$.

Direction ratios of a line

Any three numbers which are proportional to the direction cosines of a line are called the direction ratios of the line. Direction ratios of a line are denoted as *a,b,c* .

$l = ak, m = bk, n = ck$, k being a constant.

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}} m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}} n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

The sign to be taken for *l, m* and *n* depend on the desired sign of *k*, either a positive or negative.

The direction ratios of the line segment joining the points P(x_1, y_1, z_1) and Q(x_2,y_2,z_2) may be taken as $x_2–x_1$, $y_2–y_1$, $z_2–z_1$.

Equation of line in space :

1) **(a) Equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$.**

(b) Equation of a line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is $\frac{x- x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

(c) Equation of a line passing through the point (x_1, y_1, z_1) and having direction cosines l, m, n is $\frac{x- x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

2) **(a) Equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.**

(b) Equation of a line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is
$\frac{x- x_1}{x_2- x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Distance Formula :

1) **Distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by**
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2) **(a) The Shortest Distance between the Skew Lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is** $d = \left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1} X \overrightarrow{b_2})}{|(\overrightarrow{b_1} X \overrightarrow{b_2})|}\right|$

(b) The Shortest Distance between the Skew Lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ **and** $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ **is** $\boldsymbol{d} = \left|\frac{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}\right|$

3) The distance between the parallel lines $\vec{r} = \vec{a}_1 + \lambda\,\vec{b}$ **and** $\vec{r} = \vec{a}_2 + \mu\,\vec{b}$ **is** $d = \left|\frac{\vec{b}\ X(\vec{a_2}-\vec{a_1})}{|\vec{b}|}\right|$

MCQ QUESTIONS

1	If the cartesian equation of a line is $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$, write its vector equation. (a) $\vec{r} = (\hat{\imath}-4\hat{\jmath}+6\hat{k}) + \mu(5\hat{\imath}+7\hat{\jmath}+2\hat{k})$ (b) $\vec{r} = (3\hat{\imath}-4\hat{\jmath}+3\hat{k}) + \mu(-5\hat{\imath}+7\hat{\jmath}+2\hat{k})$ (c) $\vec{r} = (3\hat{\imath}+4\hat{\jmath}+3\hat{k}) + \mu(-5\hat{\imath}+7\hat{\jmath}+2\hat{k})$ (d) $\vec{r} = (3\hat{\imath}-4\hat{\jmath}+3\hat{k}) + \mu(5\hat{\imath}+7\hat{\jmath}+2\hat{k})$
2	Find the foot of the perpendicular drawn from the point (2,-3,4) on the y-axis. (a) (2,0,4) (b) (0.3.0) (c) (0,-3,0) (d) (-2,0,-4)
3	If the lines $\frac{x-2}{3}=\frac{y-1}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{-2}$ are perpendicular, find the value of **k** . (a) -2/5 (b) -2/7 (c) 4 (d) 2/7
4	The equation of a line is $\frac{x-1}{-2}=\frac{y+3}{3}=\frac{z+2}{6}$, find the direction cosines of a line parallel to the given line. (a) -2/7, 3/7, 6/7 (b) 2/7, -3/7,- 6/7 (c) -2 , 3 , 6 (d) 2 , -3 , -6

5	If a line makes angles 90^{0} and $60^{0}$with the positive direction of x and y axes ,find the angle which it makes with positive direction of z -axis. (a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) 0
6	Write direction cosines of a line parallel to z-axis. (a) 1,0,0 (b) 0,0,1 (c) 1,1,0 (d) -1,-1,-1
7	The distance of a point P (a,b,c) fom x-axis is (a) $\sqrt{a^2+b^2}$ (b) a^2+b^2 (c) a^2+c^2 (d) $\sqrt{b^2+c^2}$
8	If α,β, ϒ are the angles that a line makes with the positive direction of x,y,z axis respectively then the direction cosines of the line are (a) cosα,sinβ, cosϒ (b) cosα,cosβ, cosϒ (c) sinα,sinβ, sinϒ (d) 1, 1, 1

MCQ ANSWERS

1	(b) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu\,(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$
2	(c) (0,-3,0)
3	(a) -2/5
4	(a) -2/7, 3/7, 6/7
5	(c) $\pi/6$
6	(b) 0,0,1
7	(d) $\sqrt{b^2+c^2}$
8	(b) cosα,cosβ, cosϒ

2 MARKS QUESTIONS

1	If α,β, ϒ are the direction angles of a line, find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\Upsilon$

2	Find the direction cosines of the following line: $\frac{3-x}{-1}=\frac{2y-1}{2}=\frac{z}{4}$
3	Find the value of k so that the lines x = -y = kz and x – 2 = 2y + 1 = -z + 1 are perpendicular to each other.
4	The x –coordinate of a point on the line joining the points P (2, 2, 1) and Q (5, 1, -2) is 4 . Find its z- co ordinate
5	Check whether the lines passing through (1, 1, 2) and (3,5,1) is parallel to the line through (4, 2,-1) and (2, -2 , 0)
6	Find the vector and Cartesian equation of the line passing through points (3, -2, -5) and ,(5, -4, 6)
7	Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$
8	If the equation of a line is x = ay + b, z = cy + d, then find the direction ratios of the line and a point on the line.
9	Find the equation of a line passing through the points P(-1, 3,2) and Q(-4,2,-2).Also ,if point R(5, 5, α) is collinear with P and Q then find the value of α
10	The points A(1,2,3), B(-1,-2,-3)and C(2,3,2) are the vertices of a parallelogram,then find the equation of CD.

ANSWERS OF 2 MARKS

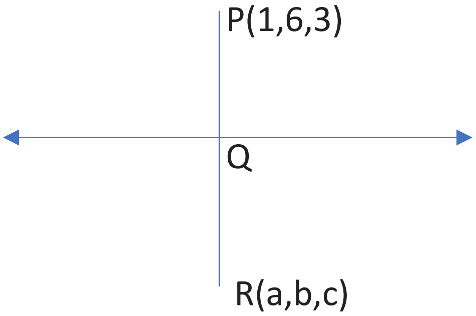
1	2
2	Direction ratios are 1 ,1 ,4 Direction cosines are $\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}$
3	The lines are : $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1/k}$ *and* $\frac{x-2}{1}=\frac{y+1}{1/2}=\frac{z-1}{-1}$ Since these lines are perpendicular $1x1+-1x\frac{1}{2}+\frac{1}{k}x-1=0$ k = 2

4	The equation of lines are $\frac{x-2}{3}=\frac{y-2}{-1}=\frac{z-1}{-3}$ Any point on this line is (3k+2, -k+2, -3k+1) 3k+2=4 $\Rightarrow$ k=2/3 z co ordinate = -3x2/3 + 1 =-2 + 1 = -1
5	The direction ratios of line joining (1, 1, 2) and (3, 5, 1) are 2,4,-1 The direction ratios of line joining (4, 2,-1)and (2, -2, 0) are -2,-4,1 Since the direction ratios are proportional they are parallel
6	i) Cartesian equation : $\frac{x-3}{2}=\frac{y+2}{-2}=\frac{z+5}{11}$ ii) Vector Equation : $\vec{a}+\lambda(\vec{b}-\vec{a})$ $(3\vec{i}-2\vec{j}-5\vec{k})+\lambda(2\hat{i}-2\hat{j}+11\vec{k})$
7	P (–2, 4, – 5) is the given point. Any point Q on the line is given by (3λ-3, 5λ + 4, 6λ-8) $\overrightarrow{PQ}=(3\lambda-1)\hat{\imath}+5\lambda\hat{\jmath}+(6\lambda-3)\hat{k}$ PQ and the given line are perpendicular $\therefore\ (3\lambda-1)3+5\lambda.5+(6\lambda-3)6=0 \quad \Rightarrow \lambda=\frac{3}{10}$ $\overrightarrow{PQ}=\frac{1}{10}\hat{\imath}+\frac{1}{10}\hat{\jmath}-\frac{12}{10}\hat{k}$ Magnitude of PQ = $\sqrt{\frac{37}{10}}$
8	Using both the conditions $\frac{x-b}{a}=y \quad \frac{z-d}{c}=y$ $\frac{x-b}{a}=\frac{y-0}{1}=\frac{z-d}{c}$ Direction ratios of the given line are a, 1, c and a point on the given line and the point on the given line is (b,0,d)
9	The equation of the line passing through P and Q is $\frac{x+1}{3}=\frac{y-3}{1}=\frac{z-2}{4}$ Since the given point is collinear with the points P and Q , R(5, 5, α) satisfies the equation Substituting in the given equation , α = 10
10	Let D(a,b,c) A (1,2,3) B(-1,-2,-3) C(2,3,2) Midpoint of AC = Midpoint of BD (3/2,5/2,5/2) = (-1+a /2, -2+b /2,-3+c /2) a=4, b=7 , c=8 D(4,7,8) Equation of CD is $\frac{x-2}{2}=\frac{y-3}{4}=\frac{z-2}{6}$

LONG ANSWER TYPE QUESTIONS

1	Find the equation of the line passing through the point P (-1,3,-2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$
2	Find the co-ordinates of the foot of the perpendicular drawn from the point A (1,8,4) to the line joining B (0, -1,3) and C (2,-3,-1).
3	(a) Find the image of the point (1,6,3) in the line: $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ (b) Also, find the length of the segment joining the given point and its image
4	Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also, find the point of intersection
5	Find the value of ,λ so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other
6	Find the equation of line passing through (1,2,3) and midpoint of the line joining (2,-1,3) and (1,2,5)
7	Show that the lines $\vec{r}=\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)+\lambda\,(3\hat{\imath}-\hat{\jmath})$ and $\vec{r}=\left(4\hat{\imath}-\hat{k}\right)+\mu\left(2\hat{\imath}+3\hat{k}\right)$ intersect each other . Find their point of intersection.
8	Find the shortest distance between the lines whose vector equations are $r^{\rightarrow}=(1-t)\,\hat{\imath}+(t-2)\,\hat{\jmath}+(3-2t)\,\hat{k}$ $\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$
9	Find the shortest distance between the lines $\vec{r}=3\tilde{\imath}+2\tilde{\jmath}-4\hat{k}+\lambda(\tilde{\imath}+2\tilde{\jmath}+2\hat{k})$ and $\vec{r}=5\tilde{\imath}-2\tilde{\jmath}+\mu(3\tilde{\imath}+2\tilde{\jmath}+6\hat{k})$ If the lines intersect find their point of intersection
10	Find the equation of the line passing through the point P (2, -1,3) and perpendicular to the lines $\vec{r}=\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)+\lambda\left(2\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}-3\hat{k}\right)+\mu\left(\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)$

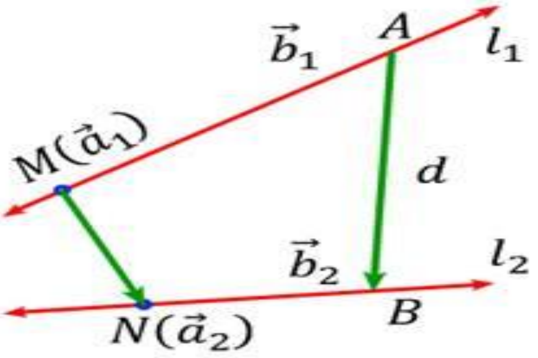
ANSWERS OF LONG ANSWER TYPE QUESTIONS

1	Equation of the line passing through P(-1,3,-2) is $\frac{x+1}{a}=\frac{y-3}{b}=\frac{z+2}{c}$ Since it is perpendicular to the two given lines a+2b+3c =0 and -3a+2b+5c =0 Solving a =2 , b=-7 , c=4 Equation of required line is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$
2	Let D be the foot of the perpendicular Equation of line BC is $\bar{r}$= -j+3k + λ (2i-2j -4k) Therefore any point D on the line is (2 λ, -1-2 λ, 3- 4 λ) Since AD is perpendicular to BC, (2 λ -1)x2 + (-1-2 λ -8)x(-2) + (3-4 λ -4)x(-4)=0 Solving we get λ = -5/6 So the required point D is (-5/3,2/3, 19/3)
3	 Q is the foot of the perpendicular from P Therefore Q is (λ ,2 λ +1,3 λ +2) PQ is perpendicular to the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$, so we have ($\lambda$ -1)x1 + (2 λ +1-6)x2 +(3 λ +2-3)x3=0 ⇒ λ = 1 Q = (1,3,5) Using midpoint formula image is (1,0,7) Required distance PR = $2\sqrt{13}$ using distance formula
4	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ ⇒ x= $2\lambda+1, y=3\lambda+2, z=4\lambda+3$ $\frac{x-4}{5}=\frac{y-1}{2}=z$ = k ⇒ x=5k+4 ,y =2k+1, z=k Lines intersect means 2 $\lambda+1=5k+4$ and $3\lambda+2=2k+1$ $\lambda=-1\ and\ k=-1$ $4\lambda+3=k$ this equation is true for $\lambda=-1\ and\ k=-1$ The lines are intersecting and the point of intersection is (-1,-1,-1)

5	$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ $\Rightarrow$ $\frac{x-1}{-3} = \frac{y-2}{2\lambda/7} = \frac{z-2}{11/5} \rightarrow (i)$ $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ $\Rightarrow$ $\frac{x-1}{-\frac{3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \rightarrow (ii)$ (i) and (ii) are perpendicular -3($-\frac{3\lambda}{7}$) +$\frac{2\lambda}{7}$ $(1) +$ $\frac{11}{5}$(-5) =0 $\Rightarrow \lambda = 7$
6	Midpoint of (2,-1,5) and (1,2,3) is ($\frac{3}{2}, \frac{1}{2}, 4)$ Dr of line joining midpoint and (1,2,3) is ½,$\frac{-3}{2}$,1 Equation of the line is $\bar{r}$ = $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ + λ($\frac{1}{2}\hat{\imath} - \frac{\hat{3}}{2}\hat{\jmath} + \hat{k}$) Cartesian equation is $\frac{x-1}{1/2} = \frac{y-2}{-3/2} = \frac{z-3}{1}$
7	The two given lines will intersect if $(\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda\,(3\hat{\imath} - \hat{\jmath}) = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ for some particular values of λ and μ Equating coefficients of $\hat{\imath}$ *and* $\hat{\jmath}$ and solving $\lambda = 1$ and $\mu = 0$ Substituting in the coefficient of $\hat{k}$, the equation is satisfied. ∴ the two lines intersect Putting $\lambda = 1$ in the first line , the point of intersection is (4,0,-1)
8	Shortest distance , d = $\frac{8}{\sqrt{29}}$
9	The shortest distance between the lines is 0 ∴The lines are intersecting Point of intersection is (−1,−6,−12)
10	Required equation of line is $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \mu\,(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$

CASE STUDY QUESTIONS

1	A student drew 2 skew lines as shown below with their points through which they pass and their directions $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$. The equations of these two lines l_1 *and*l_2 are given by$\frac{x+2}{2} = \frac{y}{3} = \frac{z-2}{1}$ and$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$



Based on the information ,answer the following

1.The vector $\overrightarrow{b_1}$ x $\overrightarrow{b_2}$ equals

(a) $5\hat{\imath} - \hat{\jmath} - 7\hat{k}$ (b) $4\hat{\imath} - 5\hat{\jmath} - 6\hat{k}$ (c)$\hat{\imath} - \hat{\jmath} + \hat{k}$(d) none of these

2.The vector $\overrightarrow{MN}$ equals

(a) $4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ (b) $5\hat{\imath} - 6\hat{\jmath} - 5\hat{k}$ (c)$5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$ (d) none of these

3.The shortest distance between these two lines $l_1\ and l_2$ is

(a) √2 units (b) √3 units (c) √5 units (d) none of these

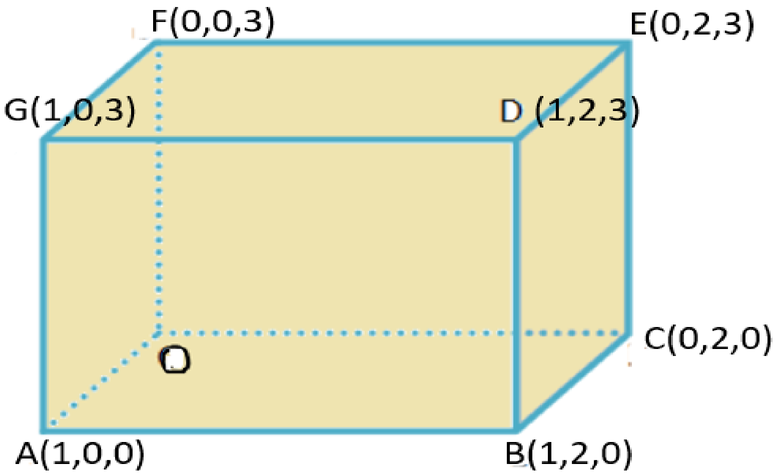
4.The lines $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{1}$are

(a)parallel (b)intersecting (c) skew lines (d) perpendicular

5. The shortest distance between the lines $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z+1}{1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{1}$is

(a) √2 units (b) √3 units (c) √5 units (d) none of these

2 Anu made a cuboidal fish tank having coordinates O (0,0,0) ,A(1,0,0), B(1,2,0) C(0,2,0) ,D(1,2,3), E0,2,3),F(0,0,3) and G(1,0,3)



Based on the above information ,answer the following questions.

1.Direction cosines of AB are

(a) < 1,1,0 > (b) < 0,2,0 > (c) <0,1,0> (d) none of these

2.Equation of diagonal OD is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$(b) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$(c) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$ (d) none of these

3. The lines BC and DE are

(a)parallel (b)intersecting (c) skew lines (d) perpendicular

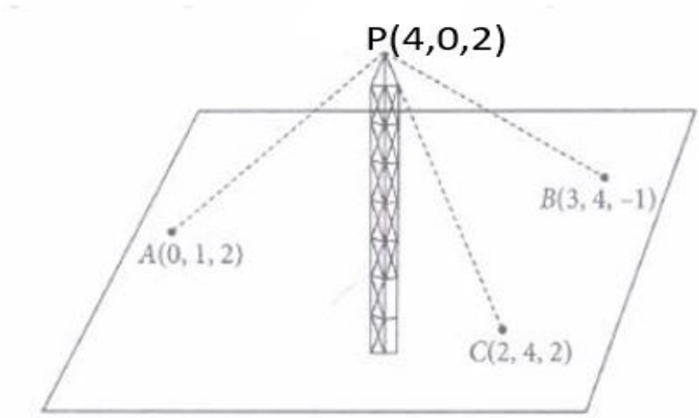
4.Equation of line BF is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-3}$ (b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (c) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$ (d) none of these

5.The lines AB and OD are

(a)parallel (b)intersecting (c) skew lines (d) perpendicular

3 A mobile tower stands at the top of a hill. Consider the surface on which tower stands as a plane having points A(0, 1,2), B(3, 4, -1) and C(2, 4, 2) on it. The mobile tower is tied with 3 cables from the point A, Band C such that it stands vertically on the ground. The peak of the tower is at the point (4,0,2), as shown in the figure.Let N(2,-3,1) be the foot of the perpendicular from the point P.



Based on the above information, answer the following questions

(i) The equation of line BC is
(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (b) $\frac{x-2}{1} = \frac{y-4}{0} = \frac{z-2}{-3}$ (c) $\frac{x-4}{3} = \frac{y}{-2} = \frac{z-2}{1}$ (d) none of these

(ii)The equation of the perpendicular line drawn from the peak of tower to the foot of the perpendicular where N(2,-3,1) is the foot of the perpendicular is

(a) $\frac{x}{3} = \frac{y}{-2} = \frac{z}{1}$ (b) $\frac{x-4}{-2} = \frac{y}{-3} = \frac{z-2}{-1}$ (c) $\frac{x-4}{3} = \frac{y}{2} = \frac{z-2}{1}$ (d) none of these

(iii) The height of the tower from the ground is

(a) 6 units (b) $\sqrt{14}$ units (c)$\sqrt{13}$ units (d) 14 units

(iv)) The direction ratios of the line joining the points A and B are
(a)1,2,3 (b) 1, 2,0 (c) 3, 3, -3 (d) 3,4 -1

(v) If Q is the reflection of the point P (4,0,2) in the line joining the points N(2,-3,1) and B(3,4,-1) ,then the coordinates of Q is

(a)(1,2,3) (b) (1, 2,0) (c) (1,2,1) (d) (0,-6,0)

ANSWERS OF CASE STUDY QUESTIONS

1	i(a) ii(c) iii(b) iv (a) v(d)
2	i(c) ii (a) iii(a) iv (a) v (c)
3	i (b) ii(b) iii (b) iv(c) v(d)

KENDRIYA VIDYALAYA SANGATHAN –ERNAKULAM REGION

STUDY MATERIAL- LINEAR PROGRAMMING

ACADEMIC YEAR 2022-23

- **A linear programming problem is one that is concerned with finding optimal value (maximum or minimum) of a linear function of several variables (called objective function) subject to the condition that the variable are non negative and satisfy a set of linear in equalities (called linear constraints). Variables are sometimes called decision variables and are non negative.**
- **The common region determined by all the constraints including the non negative constraints of a linear programming problem is called the feasible region or solution region for the problem .**
- **Points within and on the boundary of the feasible region represent feasible solution of the constraints . Any point outside the region is an infeasible solution**
- **Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called the optimal solution .**

The following Theorems are fundamental in solving linear programming problems:

Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2 Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R. If the feasible region is unbounded, then a maximum or a minimum may not exist. However, if it exists, it must occur at a corner point of R. Corner point method for solving a linear programming problem.

The method comprises of the following steps: (i) Find the feasible region of the linear programming problem and determine its corner points (vertices). (ii) Evaluate the objective function Z = ax + by at each corner point. Let M and m respectively be the largest and smallest values at these points. (iii) If the feasible region is bounded, M and m respectively are the maximum and minimum values of the objective function.

If the feasible region is unbounded, then (i) M is the maximum value of the objective function, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, the objective function has no maximum value. (ii) m is the minimum value of the objective function, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, the objective function has no minimum value.

If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type

MCQ QUESTIONS

Corner points of the feasible region for an LPP are

1 Let F = 4x+ 6y be the objective function. The Minimum value of F occurs at ....

(a) only (0, 2) (b) only (3, 0) (c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only (d) any point on the line segment joining the points (0, 2) and (3, 0).

2. Solution set of the inequality 2x+ y> 5 is ....

(a) The half plane containing origin (b) The open half plane not containing origin (c) xy- plane excepts the points on the line 2x+ y= 5 (d) None of these

3. T he point at which the maximum value of Z = 3x+ 2y subject to the constraints x+ 2y≤2, x ≥0, y ≥0 is ....

(a) (0, 0) (b) (1.5, – 1.5) (c) (2, 0) (d) (0, 2)

4. The feasible region of the inequality x+ y≤1 and x– y≤1 lies in .... quadrants.

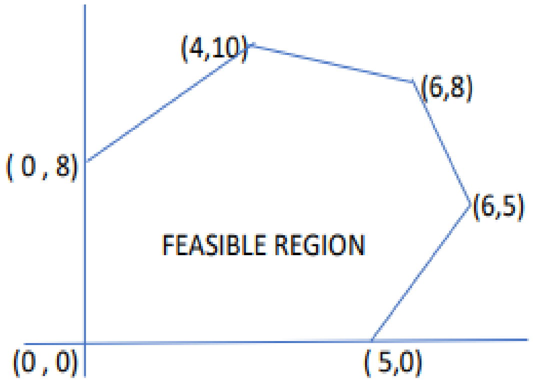
(a) Only I and II (b) Only I and III (c) Only II and III (d) All the four

5 The region represented by the in equation x – y≤ –1, x – y ≥ 0, x≥0, y≥0 is ....

(a) bounded (b) unbounded (c) do not exist (d) triangular region

SHORT ANSWER QUESTIONS

1. The feasible region of an L.P.P is shown here.If z=3x-4y is the objective function then Find the Min(Z)



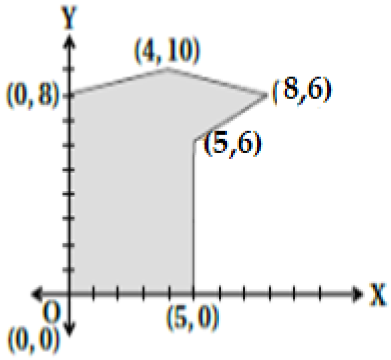
2. Find the Maximum value of Z=6X+8Y subject to the constraints 2x+y≤30,x+2y≤24,x≥0,y≥0

3. Find the point at which the maximum value of (3x + 2y) subject to the constraints x + y ≤2, x ≥ 0, y ≥ 0

4. The vertices of the feasible region determined by some linear constraints are (0, 2), (1, 1),(3, 3), (1, 5). Let Z = px+ qy where p, q> 0.Find the condition on p and q so that the maximum of Z occurs at both the points

(3, 3) and (1, 5)

5. The feasible solution for a LPP is shown in Figure Let z = 3x – 4y be the objective function.Find the points at which maximum of Z occurs



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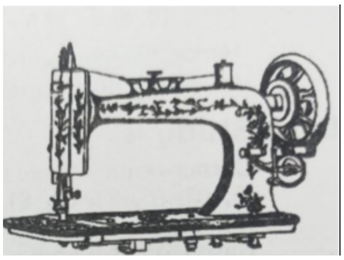
CASE STUDY QUESTIONS

1. Suppose a dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic sewing machine at a profit of Rs. 22 and a manually operated sewing machine at a profit of Rs.18. Based on the above information, answer the following question



1 Let x and y denote the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assumed that the dealer purchased atleast one of the given machines then:

(a) $x+y \geq 0$ (b) $x+y < 0$ (c) $x+y > 0$ (d) $x+y \leq 0$

Q 2 Let the constraints in the given problem is represented by the following inequalities: $x+y \leq 20$; $360x+240y \leq 5760$ and $x,y \geq 0$. Then which of the following point lie in its feasible region

a) (0,24) (b) (8,12) (c) (20,2) (d) None of these

Q 3 If the objective function of the given problem is maximize Z = 22x+18y, then its optimal value occur at: (a) (0,0))(b) (16,0) (c) (8,12) (d) (0,2

Q 4 In an LPP if the objective function Z=ax+by has the same maximum value on two corner points of the feasible region ,then the number of points at which Z_{MAX} is

a)0 b) 2 c) finite d) infinite

Q 5 If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

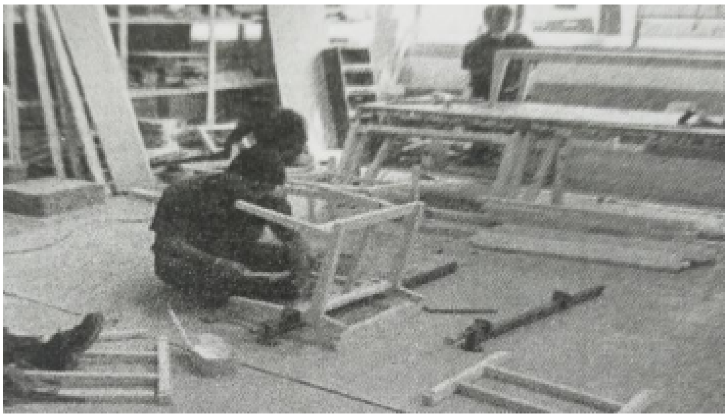
(a) The required optimal solution is at a mid pointof the line joining two points.

(b) The optimal solution occurs at every point on the line joining these two points.

(c) The LPP under consideration is not solvable.

(d) The LPP under consideration must be reconstructed

2. A manufacturing company makes two models X and Y of a product. Each piece of model X requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model Y requires 12 labour hours of fabricating and 3 labour hours for finishing, the maximum labour hours available for fabricating and finishing are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model X and Rs. 12000 on each piece of model Y. Assume x is the number of pieces of model X and y is the number of pieces of model Y. Based on the above information, answer the following questions



Q1.Which among these is not a constraint for this LPP?

(a) $9x+12y \geq 180$ (b) $3x+4y \leq 60$ (c) $x+3y \leq 30$ (d) None of these

Q2.The shape formed by the common feasible region is:

(a) Triangle (b) Quadrilateral (c) Pentagon (d) Hexagon Q

Q3.Which among these is a corner point for this LPP?

(a) (0,20) (b) (6,12) (c) (12,6) (d) (10,0)

Q4. Maximum of Z occurs at

(a) (0,20) (b) (0,10) (c) (20,10)

Q5. The sum of maximum value of Z is:

(a) 168000 (b) 16000 (c) 120000 (d) 180000

3. An aeroplane can carry a Maximum of 200 passengers.A profit of ₹1000 is made on each executive class ticket and a profit of ₹600 is made on each Economy class ticket. The airline reserves at least 20 seats for the Executive class. However at least 4 times as many passengers prefer to travel by economy class than by executive class .It is given that the number of executive class ticket is ‘x’ and that of economy class ticket isy

’.

Q1.The maximum value of $x+y$ is (A) 100 (B) 200 © 80 (D) 2

Q2.The relation between ‘x’ and ‘y’ is (A) $x < y$ (B) $x \geq 4y$ © $y > 80$ (D) $y \geq 4x$

Q3.Which among the following is not a constraint of this L.P.P (A) $x \geq 80$ (B) $x+Y \leq 20$ © $x \geq 0$ (D) $y \geq 4x$

Q4.The profit when $x=20$ and $y=80$ is ₹ (A) 60000 (B) 68000 © 64000 (D) 13600

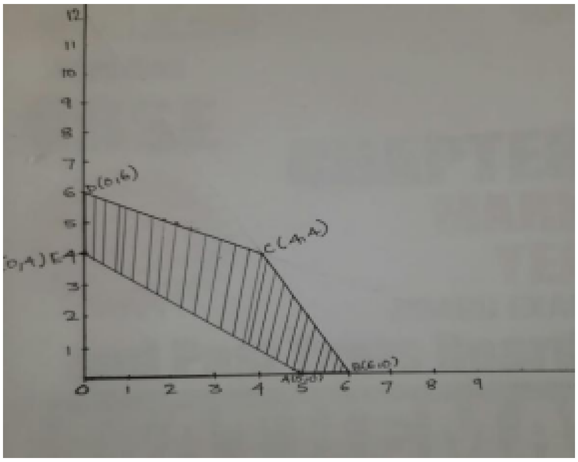
Q5.The maximum profit is

(A)128000. B)68000 C)120000 D)136000

LONG ANSWER QUESTIONS

1. Solve the following linear programming problem (L P P) graphically. Maximize $Z=2x+5y$, Subject to the constraints ; $2x+4y \leq 8$,8,$y \leq 6$6,$x+y \leq 4$ $x,y \geq 0$

2. The corner points of the feasible region determined by the system of linear constraints are as shown below



Answer each of the following (i)Let $Z =600x+400y$ be the objective function.Find the maximum and minimum value of Z and also find the corresponding points at which the maximum and minimum value occurs. (ii)Let $Z =ax + by$,where $a, b >0$ be the objective function.Find the condition on a and b so that the maximum value of Z occurs at A(2,8) and B(4,6).Also mention the number of optimal solutions in this case.

3. Minimize and maximize $Z=5x+2y$ subject to the following constraints x
$-2y \leq 2$, $3x + 2y < 12<12$ $-3x+2y\leq 3$, $x\geq 0$, $y\geq 0$

4. Minimise $(Z) = 5x + 7y$
Subject to constraints $2x + y \geq 8$, $x + 2y \geq 10$ and $x \geq 0$, $y \geq 0$

5. Maximize $Z= 3x+5y$ such that $x+3y\geq 3$, $x\geq 2$, $y\geq 0$

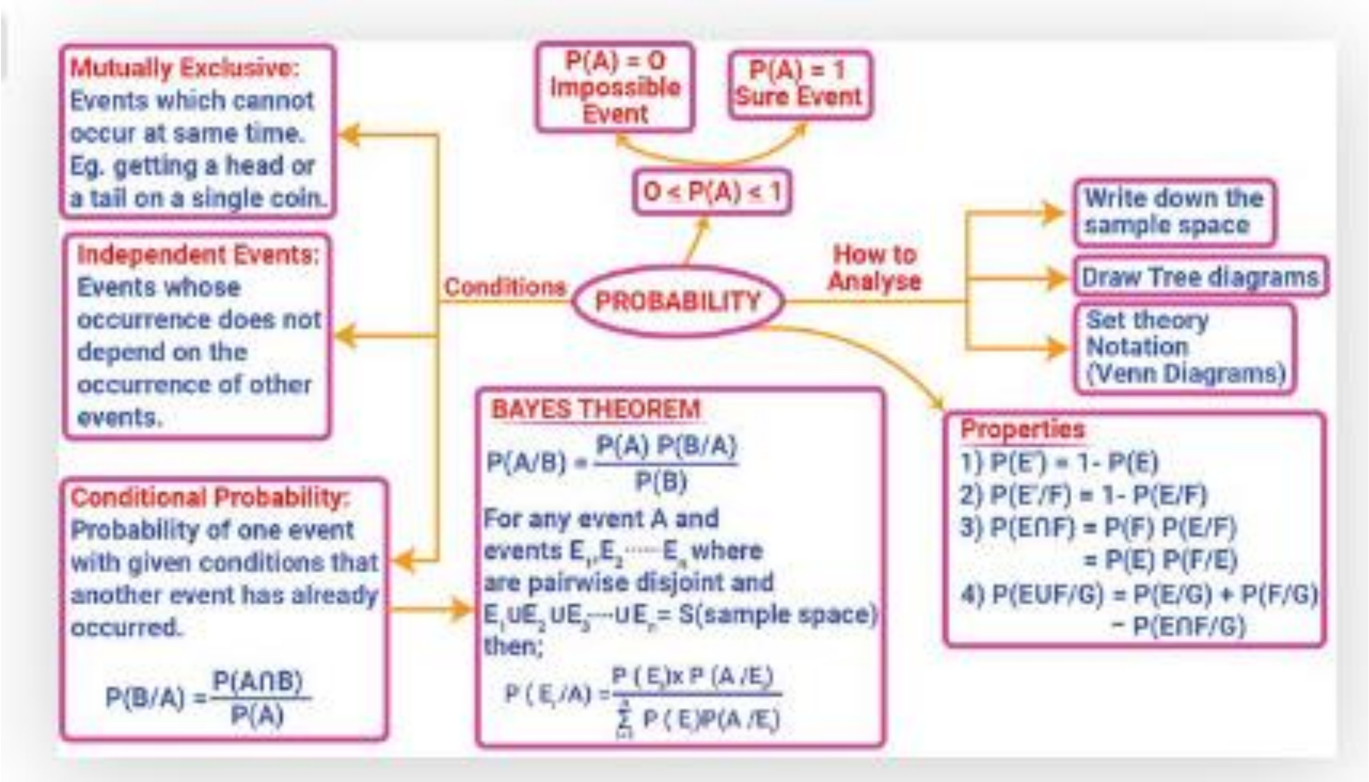
ANSWERS

MCQ	1.d	2b	3c	4c	5c
SHORT ANSWER	1)-32	2)120	3.(2,0)	4.p=q	5.(5,0)
CASE SYUDY 1	1c	2b	3c	4c	5b
CASE STUDY 2	1a	2b	3c	4d	5a
CASE STUDY 3	1b	2d	3b	4b	5d
LONG ANSWER QUESTION	1)maxZ=10	2 (i)max 3600 (ii)a=b	3) z_{max} =19 Z_{mini}=0	4)38	5) 7

CHAPTER 13

PROBABILITY

CONCEPT AND MIND MAPPING



Mutually Exclusive: Events which cannot occur at same time. Eg. getting a head or a tail on a single coin.

Independent Events: Events whose occurrence does not depend on the occurrence of other events.

Conditional Probability: Probability of one event with given conditions that another event has already occurred.

$$P(B/A) = \frac{P(A\cap B)}{P(A)}$$

BAYES THEOREM

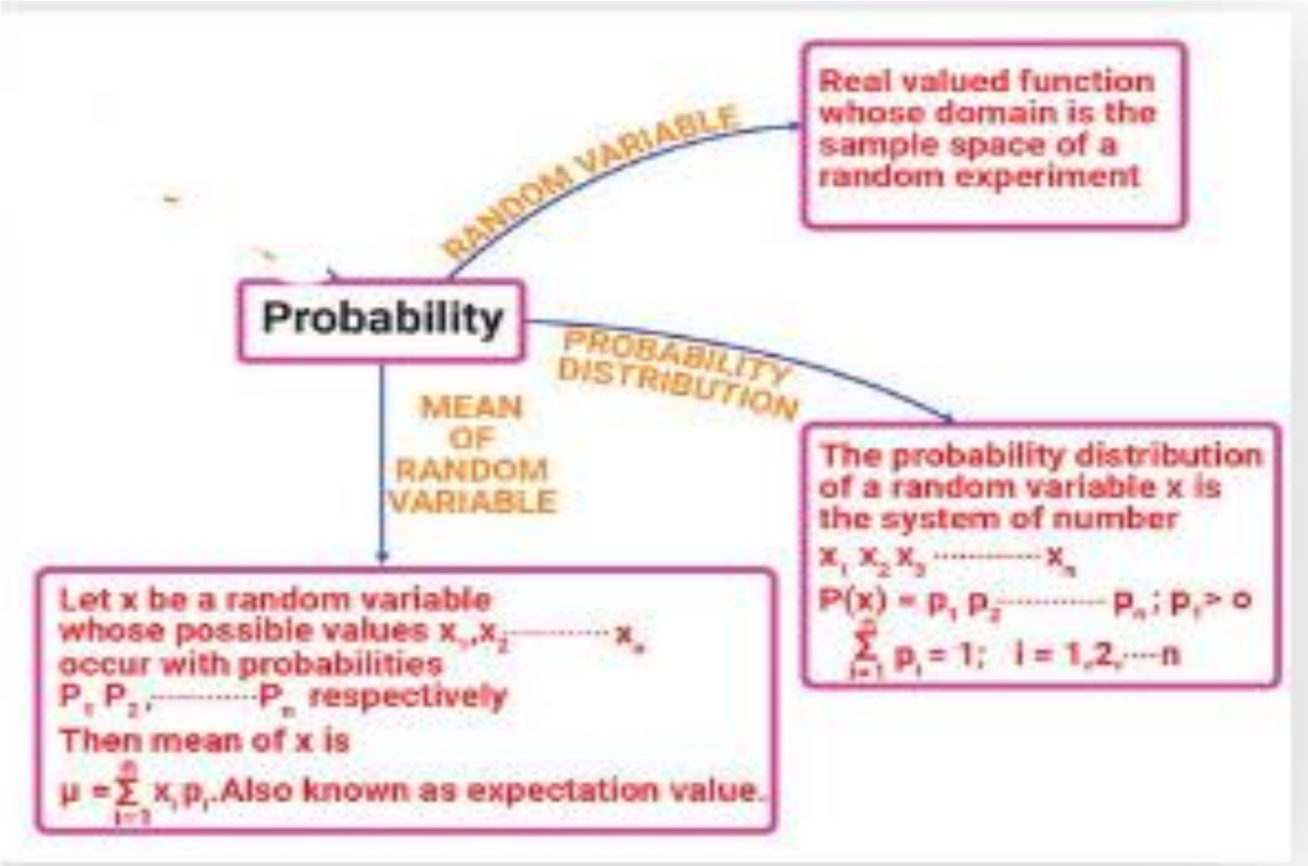
$$P(A/B) = \frac{P(A)\,P(B/A)}{P(B)}$$

For any event A and events $E_1, E_2 \cdots E_n$ where are pairwise disjoint and $E_1 \cup E_2 \cup E_3 \cdots \cup E_n = S$(sample space) then;

$$P(E_i/A) = \frac{P(E_i) \times P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

Properties

1) $P(E') = 1 - P(E)$
2) $P(E'/F) = 1 - P(E/F)$
3) $P(E \cap F) = P(F)\,P(E/F)$ $= P(E)\,P(F/E)$
4) $P(E \cup F/G) = P(E/G) + P(F/G) - P(E \cap F/G)$



Real valued function whose domain is the sample space of a random experiment

The probability distribution of a random variable x is the system of number

$x_1, x_2, x_3 \cdots\cdots\cdots x_n$

$P(x) = p_1, p_2 \cdots\cdots\cdots p_n ; p_i > 0$

$\sum_{i=1}^{n} p_i = 1; \quad i = 1, 2, \cdots n$

Let x be a random variable whose possible values $x_1, x_2 \cdots\cdots\cdots x_n$ occur with probabilities $P_1, P_2, \cdots\cdots\cdots P_n$ respectively

Then mean of x is

$\mu = \sum_{i=1}^{n} x_i p_i$. Also known as expectation value.

–

POINTS TO REMEMBER

Event: A subset of the sample space associated with a random experiment is called an event or a case.

e.g. In tossing a coin, getting either head or tail is an event.

Equally Likely Events: The given events are said to be equally likely if none of them is expected to occur in preference to the other.

e.g. In throwing an unbiased die, all the six faces are equally likely to come.

Mutually Exclusive Events: A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other, i.e. if A and B are mutually exclusive, then (A ∩ B) = Φ

e.g. In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive, since if any one of these faces comes, then the possibility of others in the same trial is ruled out.

Exhaustive Events: A set of events is said to be exhaustive if the performance of the experiment always results in the occurrence of at least one of them. If E1, E2, …, En are exhaustive events, then E1 ∪ E2 ∪……∪ En = S.

e.g. In throwing of two dice, the exhaustive number of cases is $6^2 = 36$.

Since any of the numbers 1 to 6 on the first die can be associated with any of the 6 numbers on the other die.

Complement of an Event:

Let A be an event in a sample space S, then the complement of A is the set of all sample points of the space other than the sample point in A and it is denoted by A'or $\bar{A}$.

i.e. A' = {n : n ∈ S, n ∉ A]

Note:

(i) An operation which results in some well-defined outcomes is called an experiment.

(ii) An experiment in which the outcomes may not be the same even if the experiment is performed in an identical condition is called a random experiment.

Probability of an Event:

If a trial result is n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event A, then the probability of happening of A is given by

$$P(A) = \frac{\text{Number of favourable cases to } A}{\text{Number of exhaustive cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}$$

Note: (i) $0 \leq P(A) \leq 1$

(ii) Probability of an impossible event is zero.

(iii) Probability of certain event (possible event) is 1.

(iv) P(A ∪ A’) = P(S)

(v) $P(A \cap A') = P(\Phi)$

(vi) $P(A')' = P(A)$

(vii) $P(A \cup B) = P(A) + P(B) - P(A \cap S)$

Conditional Probability:

Let E and F be two events associated with the same sample space of a random experiment. Then, probability of occurrence of event E, when the event F has already occurred, is called a conditional probability of event E over F and is denoted by P(E/F).

$$P(E/F) = \frac{P(E \cap F)}{P(F)}, \text{ where } P(F) \neq 0$$

Similarly, conditional probability of event F over E is given as

$$P(F/E) = \frac{P(F \cap E)}{P(E)}, \text{ where } P(E) \neq 0$$

Properties of Conditional Probability:

If E and E are two events of sample space S and G is an event of S which has already occurred such that $P(G) \neq 0$, then

(i) $P[(E \cup F)/G] = P(F/G) + P(F/G) - P[(F \cap F)/G]$, $P(G) \neq 0$

(ii) $P[(E \cup F)/G] = P(F/G) + P(F/G)$, if E and F are disjoint events.

(iii) $P(F'/G) = 1 - P(F/G)$

(iv) $P(S/E) = P(E/E) = 1$

Multiplication Theorem:

If E and F are two events associated with a sample space S, then the probability of simultaneous occurrence of the events E and F is

$P(E \cap F) = P(E) . P(F/E)$, where $P(F) \neq 0$ Or $P(E \cap F) = P(F) . P(F/F)$, where $P(F) \neq 0$

This result is known as multiplication rule of probability.

Multiplication Theorem for More than Two Events:

If F, F and G are three events of sample space, then

$$P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$$

Independent Events: Two events E and F are said to be independent, if probability of occurrence or non-occurrence of one of the events is not affected by that of the other. For any two independent events E and F, we have the relation

(i) $P(E \cap F) = P(F) . P(F)$

(ii) $P(F/F) = P(F), P(F) \neq 0$

(iii) $P(F/F) = P(F), P(F) \neq 0$

Also, their complements are independent events,

i.e. $P(E' \cap F') = P(E') \,.\, P(F')$

Note: If E and F are dependent events, then $P(E \cap F) \neq P(F) \,.\, P(F)$.

Three events E, F and G are said to be mutually independent, if

(i) $P(E \cap F) = P(E) \,.\, P(F)$
(ii) $P(F \cap G) = P(F) \,.\, P(G)$
(iii) $P(E \cap G) = P(E) \,.\, P(G)$
(iv) $P(E \cap F \cap G) = P(E) \,.\, P(F) \,.\, P(G)$

If at least one of the above is not true for three given events, then we say that the events are not independent.

Note: Independent and mutually exclusive events do not have the same meaning.

Baye's Theorem and Probability Distributions

Partition of Sample Space: A set of events E1, E2,…,En is said to represent a partition of the sample space S, if it satisfies the following conditions:

(i) $Ei \cap Ej = \Phi;\ i \neq j;\ i, j = 1, 2, \ldots\ldots n$
(ii) $E1 \cup E2 \cup \ldots\ldots \cup En = S$
(iii) $P(Ei) > 0,\ \forall\ i = 1, 2, \ldots, n$

Theorem of Total Probability:

Let events E1, E2, …, En form a partition of the sample space S of an experiment.

If A is any event associated with sample space S, then

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + \ldots + P(E_n)\cdot P(A/E_n) = \sum_{j=1}^{n} P(E_j)\cdot P(A/E_j)$$

Baye's Theorem:

If E1, E2,…,En are n non-empty events which constitute a partition of sample space S, i.e. E1, E2,…, En are pairwise disjoint E1 ∪ E2 ∪ ……. ∪ En = S and P(Ei) > 0, for all i = 1, 2, ….. n Also, let A be any non-zero event, the probability

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)},\ \forall\ i = 1, 2, 3, \ldots, n$$

Random Variable:

A random variable is a real-valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X.

Note: More than one random variables can be defined in the same sample space.

Probability Distributions:

The system in which the values of a random variable are given along with their corresponding probabilities is called probability distribution.

Let X be a random variable which can take n values x_1, x_2,…, x_n.

Let p_1, p_2,…, pn be the respective probabilities.

Then, a probability distribution table is given as follows:

X	x_1	x_2	x_3	……	x_n
P	p_1	p_2	p_3	……	p_n

such that $p_1 + p_2 + p_3 +… +$ pn = 1

Note: If xi is one of the possible values of a random variable X, then statement X = xi is true only at some point(s) of the sample space. Hence ,the probability that X takes value x, is always non-zero, i.e. P(X = xi) ≠ 0.

Mean of random variable

Let X be a random variable whose possible values are x_1, x_2 x_3 x_4…… …x_n occur with probabilities are p_1, p_2 p_3 p_4…… …p_n respectively . then mean of X, denoted by μ , is the number $\sum_{i=1}^{n} X_i\ p_I$ ie. The mean of X is the weighted average of the possible value of X_i , each value being weighted by its probability with which it occurs.

The mean of a random variable X is also called the expectation of X, denoted by E(X)

Mean(μ) = E(X) = μ $= \sum_{i=1}^{n}(x_i p_i) = x_1p_1 + x_2\,p_2 + \cdots . x_1p_1$ **;**

μ is called the expected value of X, ie, **E(X).**

MULTIPLE CHOICE QUESTIONS

1. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ ia equal to:
   [(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$]
2. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$ then $P(A' \cap B')$ is
   [(a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$]
3. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is
   [(a) $\frac{45}{196}$ (b) $\frac{135}{192}$ (c) $\frac{15}{56}$ (d) $\frac{15}{29}$]

4. The probability distribution of a discrete random variable X is given below

X	2	3	4	5
P(X)	5/k	7/k	9/k	11/k

   Then the value of k is
   [(a) 8 (b) 16 (c) 32 (d) 48]
5. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3 is
   [(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$]

SHORT ANSWER QUESTIONS

6. Out of 8 outstanding students of school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.
7. A fair die is rolled consider the following events A= {2,4,6}, B={4,5} and C= {3,4,5,6}.
   Find . $P[(A \cup B)/C]$
8. For the following probability distribution

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

   Find E(X)

9. The probabilities of A, B, C, solving a problem, are 1/3 , 2/7 and 3 / 8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve the problem.
10. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.
11. A coin is biased so that the head is 3 times as likely to occur as tail. If coin is tossed twice, find the probability distribution for the number of tails
12. There are two bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.

13. A four-digit number is formed using the digits 1,2,3,5 with no repetition. Find the probability that the number is divisible by 5.
14. Ten cards numbered from 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?
15. An urn contains 5 red balls, 6 green balls and 4 black balls. A ball is drawn at random from the urn. What is the probability that the ball drawn is either red or black?
16. A die is thrown twice and the sum of the numbers rising is noted to be 6. Calculate the is the conditional probability that the number 4 has arrived at least once?
17. If A and B are two independent events such that P $(\mathbf{A} \cap \mathbf{B}) = \frac{2}{15}$and P$(\mathbf{A} \cap \mathbf{B}) = \frac{1}{6}$, then find P$(\mathbf{A})$ and P$(\mathbf{B})$
18. A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?
19. From a pack of 52 playing cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?
20. A and B throw a pair of dice alternatively. A win the game if he gets a total of ‘7’ and B wins the game if he gets a total of ‘10’. If A starts the game, then find the probability that B wins.

CASE STUDY QUESTIONS

21. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $\frac{7}{9}$ be the probability that he knows the answer and $\frac{2}{9}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{9}$. Let E$_1$, E$_2$, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

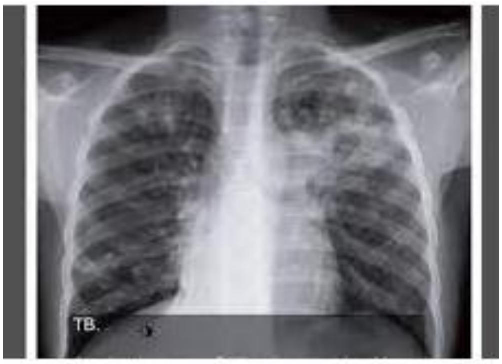
Based on the above information, answer the following:

(i) Find the value of $\sum_{k=1}^{k=2} P(E_k)$

(ii) What is the probability that the student knows the answer given that he answered it corrwectly ?

22. When a person has TB disease in lungs, the chest X-ray usually appears abnormal. By examining the chest X-ray, the probability that a person is diagnosed with TB when he is actually suffering from it is 0.99. The probability that the doctor incorrectly diagnoses a person to be having TB, on the basis of X-

ray reports is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and diagnosed to have TB.



Based on the above information, answer the following:

(i) What is the probability that the person actually having TB?

(ii) What is the probability that the person has no TB?

23. A company produing electric bulbs has factories at three location A, B, and C, and company got a bulk order of producing electric bulbs. The capacities at locations A and C are same and at loccation B is doule that of C,. Also it is known that 4% of bulbs produced at A and B are defective and 5 % produced at C are defective.

Based on the above information, answer the following questions:

(i) Find the probability of production capacity of factory at place C.

(ii) Calculate the probability of producing defective bulb.

24. A glass jar contains twenty white balls of plastic numbered from 1 to 20, ten red balls of plastic numbered from 1 to 10, forty yellow balls of plastic numbered from 1 to 40 and ten blue balls of plastic numbered from 1 to 10. If these 80 balls of plastic are thoroughly shuffled so that each ball has the same probability of being drawn.

Based on the above information answer the following:

(i) Determine the probabilities of drawing a ball of plastic that is red or yellow and numbered 1, 2, 3 or 4.

(ii) Discuss the probabilities of drawing a plastic ball which is numbered 5, 15, 25 or 35.

25. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 6%, Sonia has an error rate of 4% and Iqbal has an error rate of 3%.

Based on the above information answer the following:

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay

LONG ANSWER QUESTIONS

26. There are three coins. One is a two headed coin(having heads on both faces), another is a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coin is chosen at random and tossed , it shows heads what Is the probability , that it was the two headed coin.

27. Suppose that 5% of men and 0.25% of women have of gray hair. A gray haired person is selected at random. What is the probability of this person this being male? Assume that there are equal number of males and females.
28. A committee of 4 students is selected at random from a group consisting of 8boys and 4 girls. Given that there is at least 1 girl in the committee calculate the probability that there is exactly 2 girls in the committee.
29. Find mean (μ) for the following probability distribution.

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

30. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time. B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random. (a) What is the probability of getting a defective item? (b) If the item so chosen is found to be defective. What is the probability that it was produced by A?
31. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. (a) What is the probability that person suffers heart attach even if he has followed any of the given two options? (b) It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.
32. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces
33. Two cards are drawn from a pack of 52 cards. Find the probability distribution of number of aces
34. An Urn contains 4 white and 3 red balls. Find the probability distribution and mean of number of red balls in a random draw of three balls
35. Two cards are drawn with replacement from a pack of 52 cards. Find the probability distribution of number of kings

ANSWERS

	VERY SHORT ANSWER (MARK 1)
1	**(c)** $\because$ $P(A)=\frac{4}{5}, P(A\cap B)=\frac{7}{10}$ $\therefore$ $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$
2	**(c)** Here, $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A/B)=\frac{1}{4}$ $\because$ $P(A/B)=\frac{P(A\cap B)}{P(B)}$ $\Rightarrow$ $P(A\cap B)=P(A/B)\cdot P(B)=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$ Now, $P(A'\cap B')=1-P(A\cup B)$ $=1-[P(A)+P(B)-P(A\cap B)]$ $=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]=1-\left[\frac{6+4-1}{12}\right]$ $=1-\frac{9}{12}=\frac{3}{12}=\frac{1}{4}$
3	**(c)** Probability of getting exactly one red (R) ball $=P_R\cdot P_{\bar{R}}\cdot P_{\bar{R}}+P_{\bar{R}}\cdot P_R\cdot P_{\bar{R}}+P_{\bar{R}}\cdot P_{\bar{R}}\cdot P_R$ $=\frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{5}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{5}{6}$ $=\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}$ $=\frac{5}{56}+\frac{5}{56}+\frac{5}{56}=\frac{15}{56}$
4	**(c)** We know that, $\Sigma P(X)=1$ $\Rightarrow$ $\frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1$ $\Rightarrow$ $\frac{32}{k}=1$ $\therefore$ $k=32$
5	**(c)** Let E_1 = Event that the sum of numbers on the dice was less than 6 and E_2 = Event that the sum of numbers on the dice is 3 $\therefore$ $E_1=\{(1,4),(4,1),(2,3),(3,2),(2,2),(1,3),(3,1),(1,2),(2,1),(1,1)\}$ $\Rightarrow$ $n(E_1)=10$ and $E_2=\{(1,2),(2,1)\}\Rightarrow n(E_2)=2$ $\therefore$ Required probability $=\frac{2}{10}=\frac{1}{5}$
	SHORT ANSWERS (MARK 2)
6	**Ans:** Total number of students = 8 The number of ways to select 4 students out of 8 students $={}^8C_4=70$ The number of ways to select 2 boys and 2 girls = $={}^3C_2\times{}^5C_2=3\times10=30$ $\therefore$ Required probability $=\frac{30}{70}=\frac{3}{7}$

<table>
<tr><td>7</td><td>Given that, $A = \{2, 4, 6\}, B = \{4, 5\}, C = \{3, 4, 5, 6\}$
Now, $A \cup B = \{2, 4, 6\} \cup \{4, 5\} = \{2, 4, 5. 6\}$
So, $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$
Now, $(A \cup B) \cap C = \{2, 4, 5, 6\} \cap \{3, 4, 5, 6\} = \{4, 5, 6\}$
So, $P[(A \cup B) \cap C] = \frac{3}{6} = \frac{1}{2}$ Also $P(C) = \frac{4}{6} = \frac{2}{3}$
Required probability $= P[(A \cup B)/C] = \frac{P[(A \cup B) \cap C]}{P(C)}$
$= \frac{1/2}{2/3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$</td></tr>
<tr><td>8</td><td>$E(X) = \Sigma X\, P(X)$
$= -4 \times (0.1) + (-3 \times 0.2) + (-2 \times 0.3) + (-1 \times 0.2) + (0 \times 0.2)$
$= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$</td></tr>
<tr><td>9</td><td>25/56</td></tr>
<tr><td>10</td><td>5/17</td></tr>
<tr><td>11</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{9}{16}$</td><td>$\frac{3}{8}$</td><td>$\frac{1}{16}$</td></tr></table></td></tr>
<tr><td>12</td><td>40/61</td></tr>
<tr><td>13</td><td>1/4</td></tr>
<tr><td>14</td><td>4/7</td></tr>
<tr><td>15</td><td>3/5</td></tr>
<tr><td>16</td><td>F: Addition of numbers is 6
E: 4 has appeared at least once
So, that, we need to find P(E|F)
Finding P (E):
The probability of getting 4 atleast once is:
E = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}
Thus , P(E) = 11/ 36
Finding P (F):
The probability to get the addition of numbers is 6 is:
F = {(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)}
Thus, P(F) = 5/ 36
Also, E ∩ F = {(2,4), (4,2)}
P(E ∩ F) = 2/36
Thus, P(E|F) = (P(E ∩ F)) / (P (F))
= (2/36)/ (5/36)
Hence, Required probability is 2/5.</td></tr>
<tr><td>17</td><td>$P(A) = 5/6$ <i>and</i> $P(B) = 4/5$ OR :$P(A) = 1/5$ <i>and</i> $P(B) = 1/6$</td></tr>
<tr><td>18</td><td>B=2 /5 , A=3/5</td></tr>
<tr><td>19</td><td>13/50</td></tr>
</table>

20	$B = 5/17$
	CASE STUDY
21	**(i)** Let, E_1 = Knows the answer E_2 = guess the answer $\therefore P(E_1)=\frac{7}{9}, P(E_2)=\frac{2}{9}$ $\sum_{k=1}^{k=2} P(E_k) = P(E_1)+P(E_2) = \frac{7}{9}+\frac{2}{9} = \frac{9}{9} = 1$ **(ii)** The required probability $P\left(\frac{E}{E_1}\right) = \frac{P(E_1).P(E_1 \mid E)}{P(E_1).P(E_1 \mid E)+P(E_2)P(E \mid E_2)} = \frac{\frac{7}{9}\times 1}{\frac{7}{9}\times 1+\frac{2}{9}\times\frac{1}{9}} = \frac{\frac{7}{9}}{\frac{65}{81}} = \frac{63}{65}$
22	Ans: Let E = event that the doctor diagnoses TB, E_1 = event that the person selected is suffering from TB, and E_2 = event that the person selected is not suffering from TB. Then, $P(E_1) = \frac{1}{1000}$ and $P(E_2) = 1-\frac{1}{1000} = \frac{999}{1000}$ $P(E/E_1)$ = probability that TB is diagnosed, when the person actually has TB = $\frac{99}{100}$ $P(E/E_2)$ = probability that TB is diagnosed, when the person has no TB = $\frac{1}{1000}$ (i) Using Bayes's theorem, we have $P(E_1/E)$ = probability of a person actually having TB, if it is known that he is diagnosed to have TB = $\frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)} = \frac{\left(\frac{99}{100}\times\frac{1}{1000}\right)}{\left(\frac{99}{100}\times\frac{1}{1000}\right)+\left(\frac{1}{1000}\times\frac{999}{1000}\right)} = \frac{110}{221}$ (ii) $P(E_2/E)$ = probability of a person actually having no TB, if it is known that he is diagnosed to have TB = $\frac{P(E/E_2)P(E_2)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)} = \frac{\left(\frac{1}{1000}\times\frac{999}{1000}\right)}{\left(\frac{99}{100}\times\frac{1}{1000}\right)+\left(\frac{1}{1000}\times\frac{999}{1000}\right)} = \frac{111}{221}$
23	**Ans:** (i) Let x be the production capacity at A & C. ⇒ 2x be the production capacity at B. ∵ A : B : C = 1 : 2 : 1, Sum of production capacity = 1 + 2 + 1 = 4 If P(A), P(B) and P(C) denote the probabilities of production at places A, B and C respective Hence, $P(C) = \frac{1}{4}$ (ii) Let E be the event that bulb produced is defective Then given that $P(E/A) = 4\% = \frac{4}{100}$, $P(E/B) = 4\% = \frac{4}{100}$, $P(E/C) = 5\% = \frac{5}{100}$

<table>
<tr><td></td><td>Also $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{4} = \frac{1}{2}$, $P(C) = \frac{1}{4}$
∴ Probability that the bulb produced is defective = P(E)
∴ $P(E) = P(A) . P(E/A) + P(B) . P(E/B) + P(C) . P(E/C)$
$= \frac{1}{4} \times \frac{4}{100} + \frac{1}{2} \times \frac{4}{100} + \frac{1}{4} \times \frac{5}{100}$
$= \frac{4+8+5}{400} = \frac{17}{400}$</td></tr>
<tr><td>24</td><td>Ans: (i) Let A → event that Anand solves
B → event that Sanjay solves
C → event that Aditya solves
$P(A) = \frac{1}{2}, \ P(B) = \frac{1}{3}, \ P(C) = \frac{1}{4}$
$\therefore \ P(A') = \frac{1}{2}, \ P(B') = \frac{2}{3}, \ P(C') = \frac{3}{4}$
$P(A \cap B' \cap C') = P(A)\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$
(ii) $P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$
$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$
$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{11}{24}$</td></tr>
<tr><td>25</td><td>Ans:
$P(V) = 50\% = \frac{50}{100}, \ P(S) = 20\% = \frac{20}{100}, \ P(I) = 30\% = \frac{30}{100}$
Let E be the event that error occurred. $P(E/V) = \frac{6}{100}, \ P(E/S) = \frac{4}{100}, \ P(E/I) = \frac{3}{100}$
(i) $P(E) = P(V)\,P(E/V) + P(S)\,P(E/S) + P(I)\,P(E/I) = \frac{50}{100} \times \frac{6}{100} + \frac{20}{100} \times \frac{4}{100} + \frac{30}{100} \times \frac{3}{100}$
$= \frac{300}{10000} + \frac{80}{10000} + \frac{90}{10000} = \frac{470}{10000} = 0.047$
(ii) $P\left(\frac{V}{E}\right) = \frac{P(V)P\left(\frac{E}{V}\right)}{P(E)} = \frac{50/100 \times 6/100}{47/1000} = \frac{30}{47}$
∴ $P(\text{Not processed by Vinya}) = 1 - \frac{30}{47} = \frac{17}{47}$.</td></tr>
<tr><td colspan="2">LONG ANSWER</td></tr>
<tr><td>26</td><td>4/9</td></tr>
<tr><td>27</td><td>20/21</td></tr>
<tr><td>28</td><td>168/425</td></tr>
<tr><td>29</td><td>Mean(μ)= 3/2</td></tr>
<tr><td>30</td><td>(a) 34 /1000 (b) 5/ 34</td></tr>
<tr><td>31</td><td>(a) 29/ 100 (b) 14 /29</td></tr>
<tr><td>32</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{188}{221}$</td><td>$\frac{32}{221}$</td><td>$\frac{1}{221}$</td></tr></table></td></tr>
</table>

33

$$P(X=0)=\frac{48C_2}{52C_2}=\frac{48\times47}{52\times51}=\frac{188}{221}$$

$$P(X=1)=\frac{4C_1\times48C_1}{52C_2}=\frac{48\times4\times2}{52\times51}=\frac{32}{221}$$

$$P(X=2)=\frac{4C_2}{52C_2}=\frac{4\times3}{52\times51}=\frac{1}{221}$$

X	**0**	**1**	**2**
P(X0)	**188/221**	**32/221**	**1/221**

34

Let X be the no. of red balls in a random draw of three bal

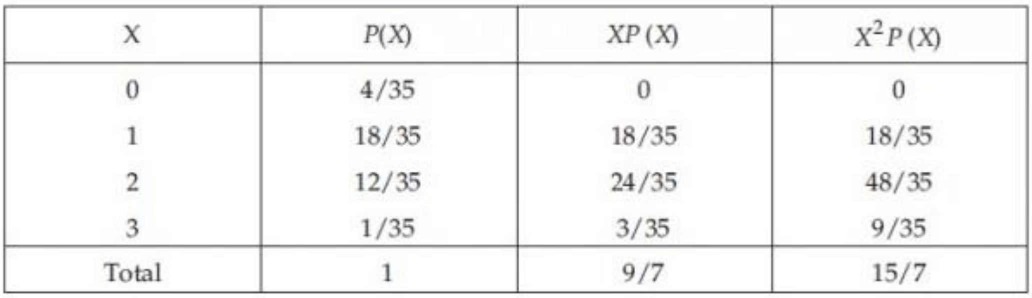
As there are 3 red balls, possible values of X are 0, 1, 2, 3.

$$P(0)=\frac{{}^3C_0\times{}^4C_3}{{}^7C_3}=\frac{4\times3\times2}{7\times6\times5}=\frac{4}{35}$$

$$P(1)=\frac{{}^3C_1\times{}^4C_2}{{}^7C_3}=\frac{3\times6\times6}{7\times6\times5}=\frac{18}{35}$$

$$P(2)=\frac{{}^3C_2\times{}^4C_1}{{}^7C_3}=\frac{3\times4\times6}{7\times6\times5}=\frac{12}{35}$$

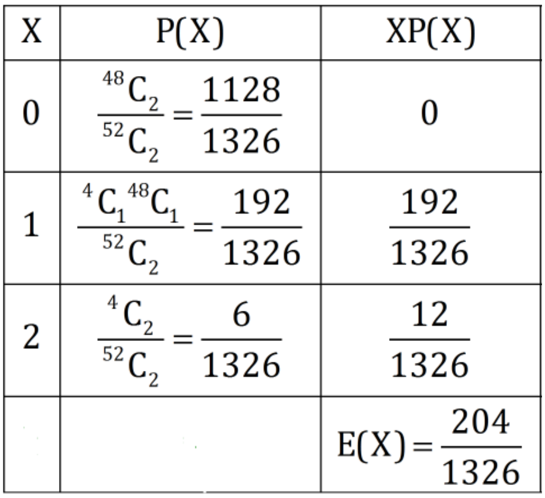
$$P(3)=\frac{{}^3C_3\times{}^4C_0}{{}^7C_3}=\frac{1\times1\times6}{7\times6\times5}=\frac{1}{35}$$



X	$P(X)$	$XP(X)$	$X^2P(X)$
0	4/35	0	0
1	18/35	18/35	18/35
2	12/35	24/35	48/35
3	1/35	3/35	9/35
Total	1	9/7	15/7

$$\text{Mean}=\Sigma XP(X)=\frac{9}{7}$$

35



X	P(X)	XP(X)
0	$\frac{^{48}C_2}{^{52}C_2} = \frac{1128}{1326}$	0
1	$\frac{^{4}C_1 \, ^{48}C_1}{^{52}C_2} = \frac{192}{1326}$	$\frac{192}{1326}$
2	$\frac{^{4}C_2}{^{52}C_2} = \frac{6}{1326}$	$\frac{12}{1326}$
		$E(X) = \frac{204}{1326}$

Appendix -1

CBSE - Sample Question Paper
Class XII
Session 2022-23
Mathematics (Code-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each**)** with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 mark

Q1. If A =[a_{ij}] is a skew-symmetric matrix of order n, then

(a) $a_{ij} = \frac{1}{a_{ji}} \ \forall\ i,j$ (b) $a_{ij} \neq 0 \ \forall\ i,j$ (c)$a_{ij} = 0, where\ i = j$ (d) $a_{ij} \neq 0\ where\ i = j$

Q2. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$

(a) 9 (b) -9 (c) 3 (d) -3

Q3. The area of a triangle with vertices A, B, C is given by

(a) $|\overrightarrow{AB} \times \overrightarrow{AC}|$ (b) $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

(b) $\frac{1}{4}|\overrightarrow{AC} \times \overrightarrow{AB}|$ (d) $\frac{1}{8}|\overrightarrow{AC} \times \overrightarrow{AB}|$

Q4. The value of 'k' for which the function f(x) =$\begin{cases} \frac{1-cos4x}{8x^2}, if\ x \neq 0 \\ k, if\ x = 0 \end{cases}$ is continuous at x = 0 is

(a) 0 (b) -1 (c) 1. (d) 2

Q5. If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is

(a) $x^2 + \log|x| + C$ (b) $\frac{x^2}{2} + \log|x| + C$ (c)$\frac{x}{2} + \log|x| + C$ (d) $\frac{x}{2} - \log|x| + C$

Q6. If m and n, respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then m + n =

(a) 1 (b) 2 (c) 3 (d) 4

Q7. The solution set of the inequality 3x + 5y < 4 is

(a) an open half-plane not containing the origin.
(b) an open half-plane containing the origin.
(c) the whole *XY*-plane not containing the line 3x + 5y = 4.
(d) a closed half plane containing the origin.

Q8. The scalar projection of the vector $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ $on\ the\ vector$ $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is

(a) $\frac{7}{\sqrt{14}}$ (b) $\frac{7}{14}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$

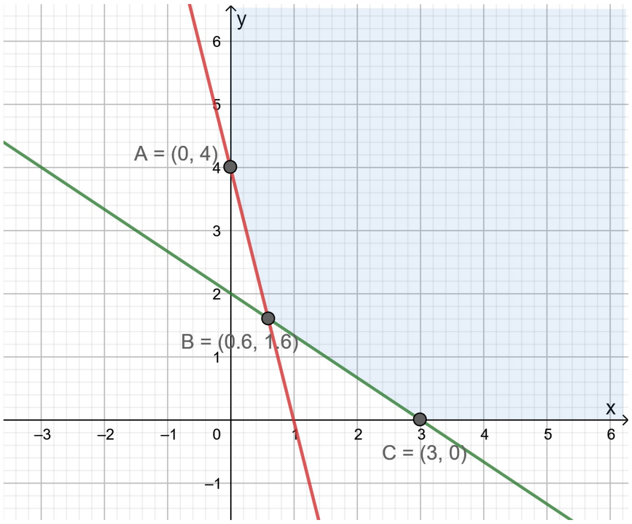
Q9. The value of $\int_2^3 \frac{x}{x^2+1}$dx is

(a) log4 (b) $log\frac{3}{2}$ (c) $\frac{1}{2}log2$ (d) $log\frac{9}{4}$

Q10. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$

(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

Q11. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at



(a) $(0.6, 1.6)$ $only$ (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only

(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

Q12. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, $then\ the\ possible$ value(s) of ‘x’ is/are

(a) 3 (b) $\sqrt{3}$ (c) -$\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

Q13. If A is a square matrix of order 3 and |A| = 5, then $|adjA| =$

(a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$

Q14. Given two independent events A and B such that P(A) =0.3, P(B) = 0.6 and P$(A' \cap B')$ is

(a) 0.9 (b) 0.18 (c) 0.28 (d) 0.1

Q15. The general solution of the differential equation $ydx - xdy = 0\ is$

(a) $xy = C$ (b) $x = Cy^2$ (c) $y = Cx$ (d) $y = Cx^2$

Q16. If $y = sin^{-1}x$, then $(1 - x^2)y_2\ is$ equal to

(a) xy_1 (b) xy (c) xy_2 (d) x^2

Q17. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4, then$ $|\vec{a} - 2\vec{b}|$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$

Q18. P is a point on the line joining the points $A(0,5,-2)$ and $B(3,-1,2)$. If the x-coordinate of P is 6, then its z-coordinate is

(a) 10 (b) 6 (c) -6 (d) -10

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A)**: The domain of the function $sec^{-1}2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup [\frac{1}{2}, \infty)$

Reason (R): $sec^{-1}(-2) = -\frac{\pi}{4}$

Q20. **Assertion (A)**: The acute angle between the line $\bar{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - \hat{\jmath})$ and the x-axis is $\frac{\pi}{4}$

Reason(R): The acute angle θ between the lines

$\bar{r} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k} + \lambda(a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k})$ and

$\bar{r} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k} + \mu(a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k})$ is given by $cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}$

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

Q21. Find the value of $sin^{-1}[sin\left(\frac{13\pi}{7}\right)]$

OR

Prove that the function f is surjective, where $f: N \to N$ such that

$$f(n) = \begin{cases} \dfrac{n+1}{2}, if\ n\ is\ odd \\ \dfrac{n}{2}, if\ n\ is\ even \end{cases}$$

Is the function injective? Justify your answer.

Q22. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

Q23. If $\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

OR

Find the direction ratio and direction cosines of a line parallel to the line whose equations are

$6x - 12 = 3y + 9 = 2z - 2$

Q24. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, $then\ prove\ that\ \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

Q25. Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q26. Find: $\int \frac{dx}{\sqrt{3-2x-x^2}}$

Q27. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made?

OR

Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size.

Q28. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$

OR

Evaluate: $\int_0^4 |x-1|\, dx$

Q29. Solve the differential equation: $ydx + (x - y^2)dy = 0$

OR

Solve the differential equation: $xdy - ydx = \sqrt{x^2 + y^2}\, dx$

Q30. Solve the following Linear Programming Problem graphically:

Maximize Z = 400x + 300y subject to $x + y \leq 200, x \leq 40, x \geq 20, y \geq 0$

Q31. Find $\int \frac{(x^3+x+1)}{(x^2-1)} dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Q32. Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration.

Q33. Define the relation R in the set $N \times N$ as follows:

For (a, b), (c, d) ∈ $N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$.

OR

Given a non-empty set X, define the relation R in P(X) as follows:
For A, B $\in P(X), (A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

Q34. An insect is crawling along the line $\bar{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and another insect is crawling along the line $\bar{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest? Find the shortest possible distance between them.

OR

The equations of motion of a rocket are:
$x = 2t, y = -4t, z = 4t,$ where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point O(0, 0, 0) and from the following line in 10 seconds?
$\vec{r} = 20\hat{\imath} - 10\hat{\jmath} + 40\hat{k} + \mu(10\hat{\imath} - 20\hat{\jmath} + 10\hat{k})$

Q35. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations
$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

Q36. **Case-Study 1**: Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = -0.1x^2 + mx + 98.6, 0 \le x \le 12$, m being a constant, where f(x) is the temperature in °F at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.
(ii) If 6 is the critical point of the function, then find the value of the constant m.

(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

OR

(iii) Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

Q37. **Case-Study** 2: Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

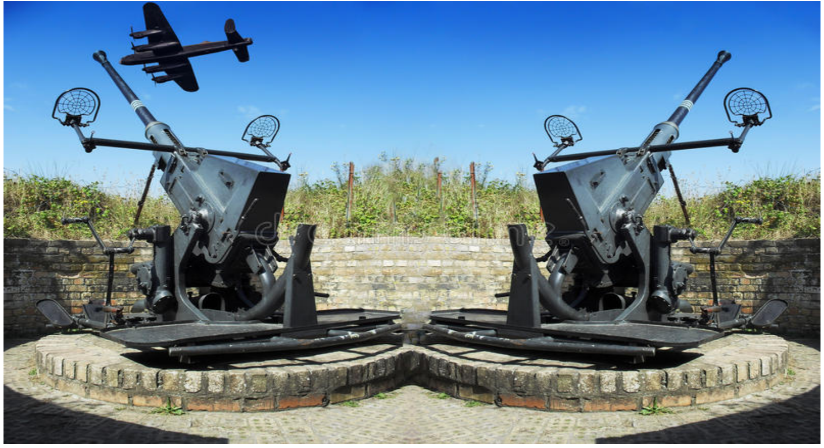
(ii) Find the critical point of the function.

(iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

Q38. **Case-Study** 3: Read the following passage and answer the questions given below.



There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

Appendix - 2

KENDRIYA VIDYALAYA ERNAKULAM REGION

BLUE PRINT FOR SAMPLE PAPER

SUBJECT :Mathematics

	Name of Chapters	Section A		Section B	Section C	Section D	Section E		Total
		MCQ	ARQ	VSA	SA	LA	CBQ		
1	Relations and Functions			2(1)*		5(1)*			7(2)
2	Inverse trigonometric Functions		1(1)						1(1)
3	Matrices	1(1)							1(1)
4	Determinants	4(4)				5(1)			9(5)
5	Continuity and Differentiability	2(2)		2(1)					4(3)
6	Application of Derivatives			2(1)			4(1)		6(2)
7	Integrals	2(2)			9(3)*				11(5)
8	Application of Integrals					5(1)	4(1)		9(2)
9	Differential equations	2(2)			3(1)*				5(3)
10	Vectors	3(3)		4(2)*					7(5)
11	3 Dimensional Geometry	1(1)	1(1)			5(1)*			7(3)
12	Linear Programming	2(2)			3(1)				5(3)
13	Probability	1(1)			3(1)*		4(1)		8(3)
		18(18)	2(2)	10(5)	18(6)	20(4)	12(3)		80(38)

*Internal choice

CLASS XII SMPLE PAPER 1

SUBJECT :MATHEMATICS

Time Allowed :3 hours **Maximum marks :80**

General instructions

1.The question paper contains -five sections A <B<C<D and Each section is compulsory .However there are internal choices in some questions.

2.Section A has i8 MCQ'S and 2 Assertion –Reason based questions of 1 mark each

3.Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.

4. Section C has 6 Short Answer (SA) type questions of 3 marks each.

5. Section D has 4 Long Answer (LA) type questions of 25 marks each.

6.Section E has 3 source based /case based /passage based /integrated units of assessment(4 marks)each with sub parts

SECTION A

(Multiple choice questions) (Each questions carries 1 mark)

1. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$ find the value of $x+y$.

A. 10 B. 2 C. -1 D. 1

2. If $\begin{vmatrix}3 & x\\x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\4 & 1\end{vmatrix}$ then find the value of x.

A. $\pm 2\sqrt{2}$ B.$\pm 2\sqrt{3}$ C.$\pm\sqrt{14}$ D. $\pm\sqrt{2}$

3. Let A be a square matrix of order 3 and $|A|=4$, then the write the value of $|AdjA|$.

A. 4 B. 16 C. 32 D.8

4. The area of a triangle with vertices $(2,-6)$, $(5,4)$ and $(k,4)$ is 35 square units then , k is

A. 12 B. -2 C. $-12,-2$ D. $12,-2$

5. $A=\begin{bmatrix}2 & -3 & 5\\6 & 0 & 4\\1 & 5 & -7\end{bmatrix}$,then find the value of $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}$ where A_{ij} is the cofactor of a_{ij}.

A. 28 B.-28 C.0 D.-24

6. Find the value of k so that the function $f(x)$ = $\begin{cases} kx^2 & ; x \le 2 \\ 3 \; ; & x > 2 \end{cases}$ is continuous at $x=2$.

A. 3 B. $\frac{3}{4}$ C. $\frac{4}{3}$ D.2

7. If $y=\ cos\sqrt{x}$ find $\frac{dy}{dx}$.

A. $\frac{sin\sqrt{x}}{2\sqrt{x}}$ B. $\frac{-sin\sqrt{x}}{2\sqrt{x}}$ C. $\frac{sin\sqrt{x}}{\sqrt{x}}$ D. $-sin\sqrt{x}$

8. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\, dx$ is

A. $e^{\tan^{-1}x} + C$ B. $\log|e^{\tan^{-1}x}| + C$ C. $\frac{(\tan^{-1}x)^2}{2} + C$ D. $\frac{1}{\tan^{-1}x} + C$

9. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + xcosx + tan^5x + 1)\, dx$

A. 2 B. π C. 1 D. 0

10. The degree of the differential equation $x^2\frac{d^2y}{dx^2} = (x\frac{dy}{dx} - y)^3$ is

A. 2 B. 3 C. 1 D. 6

11. The general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is

A. $y = \frac{x^2+C}{4x^4}$ B. $y = \frac{x^4+C}{4x}$ C. $y = \frac{x^4+C}{4x^2}$ D. $\frac{x^2}{4} + C$

12. If $|\vec{a}| = 10$, $|\vec{b}| = 2$; $\vec{a}.\vec{b}$ = 12 ,the value of $|\hat{a} \times \hat{b}|$ is

A. 5 B. 10 C. 14 D. 16

13. The vector having initial and terminal points as (2,5,0) and (-3,7,4) respectively is

A. $5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ B. $-\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ C. $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ D. $-5\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

14. The area of triangle formed by the vertices O,A,B where $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ is

A. $3\sqrt{5} sq\ units$ B. $5\sqrt{5} sq\ units$ C. $6\sqrt{5} sq\ units$ D. $4\ sq\ units$

15. If a line makes an angle $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with X axis and Y axis respectively , then the angle made by the line with z axis is

A. $\frac{\pi}{3}$ B. $\frac{\pi}{2}$ C. $\frac{\pi}{4}$ D. $\frac{5\pi}{4}$

16. The corner points of the feasible region determined by the system of linear constraints are $(0,3)$ $(1,1)$ and (3,0).Let $Z = px + qy$; $p, q > 0$.Condition on p and q so that minimum of Z occurs at (3,0) and (1,1) is

A. $p = 2q$ B. $p = \frac{q}{2}$ C. $p = 3q$ D. $p = q$

17. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 10, x, y \geq 0$ is

A. 36 B. 40 C. 20 D. 15

18. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$, $P(A \cup B) = 0.5$,then $P(A \cap B')$ is

A . $\frac{2}{3}$ B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{5}$

ASSERTION- REASON BASED QUESTIONS

In the following questions, a statement of assertion (a) is followed by a statement of Reason.

Choose the correct answer out of the following choices.

A .Both A and R are true and R is the correct explanation of A

B. Both A and R are true and R is not the correct explanation of A

C. A is true but R is false

D. A is false but R is true

19. Assertion(A): Range of $\tan^{-1} x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$

Reason (R):Domain of $\tan^{-1} x$ is R

20. Assertion(A):Pis a point on the line segment joining the points $(3,2,-1)$ and $(6,-4,-2)$.If x coordinate of P is 5 ,then its y coordinate is -2.

Reason (R): The two lines $x = ay + b$, $z = cy + d$ and lines $x = a'y + b'$, $z = c'y + d'$ will be perpendicular iff $aa' + bb' + cc' = 0$

SECTION B

(This section comprises of very short answer type questions(VSA) of 2 marks each.)

21. A ladder 5m long is leaning against a wall .The bottom of the ladder is pulled along the ground away from the wall at the rate of 2cm/s How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

22. Find $\frac{dy}{dx}$ at $t = \frac{\pi}{3}$ when $x = 10(t - sint)$; $y = 12(1 - cost)$

23 .Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ be two vectors ,show that ($\vec{a}$+$\vec{b}$)and ($\vec{a}$-$\vec{b}$) are perpendicular to each other.

24. Let $A = B = \{x: x\epsilon R \ ; \ -1 \leq x \leq 1\}$. show that the function $f : A \rightarrow B$ given by $f(x) = x|x|$ is bijective

OR

What is the principal value of $\cos^{-1}(\cos\frac{2\pi}{3}) + (\sin^{-1}(\sin\frac{2\pi}{3})$

25. Find the equation of the line in vector and Cartesian form that passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and in the direction of $\hat{\imath} + 2\hat{\jmath} - \hat{k}$.

OR

If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ represent two adjacent sides of a parallelogram,find unit vectors parallel to the diagonals of the parallelogram.

SECTION C

(This section comprises of short answer type questions(SA) of 3 marks each.

26 . Evaluate $\int \frac{x^2}{1-x^4}\,dx$

OR

Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{cos^3x\sqrt{2sin2x}}\,dx$

27. Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$

28. Evaluate $\int_{-1}^{2} |x^3 - x|dx$

29. Solve the differential equation $\frac{dy}{dx} + 2ytanx = sinx$ given that $y = 0$ when $x = \frac{\pi}{3}$

OR

Solve the differential equation $(x+y)(dx-dy) = dx + dy$

30. Maximize $Z = 300x + 190y$ subject to the constraints $x + y \leq 24$; $x + \frac{1}{2}y \leq 16$; $x, y \geq 0$.

31. A refrigerator contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random , find the probability distribution of number of milk chocolates.

OR

The probability of two students A and B coming to school are $\frac{2}{7}$ and $\frac{4}{7}$ respectively. Assuming that the events' A coming on time' and 'B coming on time are independent' ,find the probability of only one of them coming to school on time.

SECTION D

(This section comprises of long answer type questions(LA) of 5 marks each.

32 Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

33 . Show that the relation R in set $A = \{ x \in Z, 0 \leq x \leq 12\}$ given by

$R = \{(a,b): |a-b| \; is \; a \; multiple \; of \; 4\}$ is an equivalence relation

34 Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle

$x^2 + y^2 = 32.$

35 Find shortest distance between two skew lines

$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

SECTION E

This section comprises of 3 case study /passage based questions of 4marks ech with two subparts .First two case study questions have three sub-parts (i), (ii), (iii) of marks 1,1,2 respectively.The third case study questions has two sub parts of 2 marks each.

36 Shreya got a rectangular parallelepiped shaped box and spherical ball inside it as return gift ,sides of the box are x, $2x$, and $\frac{x}{3}$,while radius of the ball is r

Based on the above information ,answer the following questions

(i) If S represents the sum of volume of parallelepiped and sphere, then find S
(ii) If the sum of the surface areas of box and ball are given to be constant k^2, then find x
(iii) Calculate radius r when S is minimum

37. A student Arun is running on a playground along the curve given by $y = x^2 + 7$, another student Manita standing at point $(3,7)$ on playground wants to hit Arun by paper ball when Arun is nearest to Manita

Based on the above information answer the following questions

(i) what is Arun's position at any value of x
(ii) Find distance between Arun and Manita
(iii) Find position of Arun when Manita will hit the paper ball

38 . The equation of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time t is given in seconds and the distance is measured in kilometers.

i) At what distance will the rocket be from the starting point $(0,0,0)$ in 5 seconds?
ii) If the position of the rocket at a certain instant of time is(5,-8,10),then what will be the height of the rocket from the ground?(ground is considered as the xy plane.

……………………………………………………………….

ANSWER KEY

1. (C) -1
2. (A) $x = \pm 2\sqrt{2}$
3. (B) 16
4. (D) 12,-2
5. (C) 0
6. (B) $k = \frac{3}{4}$
7. (B) $\frac{dy}{dx} = \frac{-sin\sqrt{x}}{2\sqrt{x}}$.
8. (A) $e^{\tan^{-1}x} + C$
9. (B) π
10. (C) 1
11. $(C\ )y = \frac{x^4+C}{4x^2}$
12. $(D)16$
13. (C) $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$
14. $(A)3\sqrt{5}sq\ units$
15. (A) $\frac{\pi}{3}$
16. $(B)p = \frac{q}{2}$
17. $(B)40$
18. (D) $\frac{1}{5}$
19. (B) Both A and R are true and R is not the correct explanation of A
20. (c) A is true but R is false

SECTION B

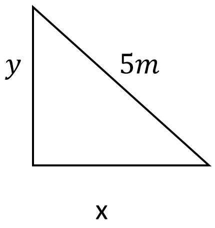
21. $\frac{dx}{dt} = 2\ cm/s$

$x^2 + y^2 = 25$….(1)

when $x = 4m$ $y = 3cm$ (½)

Differentiating with respect to t $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ (1)

$\frac{dy}{dt} = -\frac{8}{3}cm\ /s$ (½)



Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{8}{3}cm\ /s$.

22 $x = 10(t - sint);$ $\frac{dx}{dt} = 10(1 - cost)$ ½

$y = 12(1 - cost);$ $\frac{dy}{dt} = 12\ sint$ ½

$\frac{dy}{dx} = \frac{12sint}{10(1-cost)}$ ½ at $t = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{6\sqrt{3}}{5}$ ½

23 ($\vec{a}$+$\vec{b}$)= $4\,\hat{\imath} + \hat{\jmath} - \hat{k}$ ½

($\vec{a}$-$\vec{b}$) =$-2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ ½

($\vec{a}$+$\vec{b}$) . ($\vec{a}$-$\vec{b}$) = $-8 + 3 + 5 = 0$ 1

Therefore ($\vec{a}$+$\vec{b}$)and ($\vec{a}$-$\vec{b}$) are perpendicular to each other.

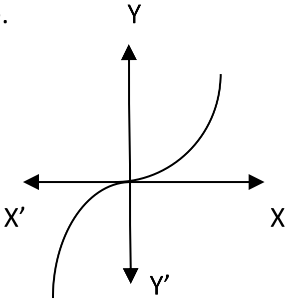
24 $f(x) = x|x| = \begin{cases} x^2 \;;\; x \geq 0 \\ -x^2 \;;\, x < 0 \end{cases}$ ½

For $x \geq 0$, $f(x) = x^2$ represents a parabola opening upward and for $x < 0$, $f(x) = -x^2$ a parabola opening down ward . ½

Since any line parallel to x axis will cut the graph at only one point f is one –one ½

Also any line parallel to y axis will cut the graph , f is on to. ½

So f is bijective.



OR

$\cos^{-1}(\cos\frac{2\pi}{3}) + (\sin^{-1}(\sin\frac{2\pi}{3})$= $\frac{2\pi}{3} + \sin^{-1}(\sin\pi - \frac{\pi}{3})$ 1

= $\frac{2\pi}{3} + \frac{\pi}{3}$ ½

$= \pi$ ½

25. $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$

$\widehat{\vec{b} = \imath} + 2\hat{\jmath} - \hat{k}$. ½

Vector equation of the line passing through the point having position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \mu\vec{b}$

$\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ ½

Cartesian equation is $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1} = \mu$ 1

OR

$\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}\ , \vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$

Then the diagonal of the parallel gram is given by $\vec{a} + \vec{b}$

$\vec{a} + \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} = 3\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$ (1)

Unit vector parallel to the diagonal is $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{9+36+4}}$ ½

$= \frac{3\hat{\imath}+6\hat{\jmath}-2\hat{k}}{7}$ ½

SECTION C

26. $\int \frac{x^2}{1-x^4}\,dx = \int \frac{\frac{1}{2}+\frac{x^2}{2}-\frac{1}{2}+\frac{x^2}{2}}{(1-x^2)((1+x^2)}\,dx$ ½

$= \int \frac{\frac{1}{2}(1+x^2)}{(1-x^2)((1+x^2)}\,dx - \int \frac{\frac{1}{2}(1-x^2)}{(1-x^2)((1+x^2)}\,dx$ ½

$= \frac{1}{2}\int \frac{1}{(1-x^2)}\,dx - \frac{1}{2}\int \frac{1}{(1+x^2)}\,dx$ 1

$= \frac{1}{4}\log\left|\frac{1+x}{1-x}\right| + \frac{1}{2}\tan^{-1}x + C$ 1

OR

$I = \int_0^{\frac{\pi}{4}} \frac{1}{cos^3x\sqrt{2sin2x}}\ dx = \int_0^{\frac{\pi}{4}} \frac{1}{cos^3x\sqrt{4\ sinxcosx}}\ dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{cos^3x\sqrt{\frac{sinx}{cosx}}\ cos^2x}\ dx$ ½

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{cos^4x\sqrt{tanx}}\ dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{sec^2x\ sec^2x}{\sqrt{tanx}}\ \ dx$ 1

Let $tanx = t\ \ ;\ sec^2x\ dx = dt$

$x = 0\ \ then\ t = 0\ \ ;\ \ x = \frac{\pi}{4}\ \ then\ t = 1$ ½

$$I = \frac{1}{2}\int_0^{`1} \frac{(1+t^2)}{\sqrt{t}}\ \ dt$$

$= \frac{6}{5}$ 1

27. $\int \frac{\sin(x-a)}{\sin(x+a)}\ dx$

$x + a = t\ \ ;\ \ dx = dt$ ½

$\int \frac{\sin(x-a)}{\sin(x+a)}\ dx = \int \frac{\sin(t-2a)}{\sin \mathrm{t}}\ dx$ ½

$= \int \frac{sint\ cos2a - cost\ sin2a}{sint}\ \ dx$ ½

$= \cos 2a\ \int dt - sin2a \int \cot t\ dt$

$= cos2a\ .t - sin2a \log|sint| + C$ 1

$= cos2a\ (x+a) - sin2a \log|\sin(x+a)| + C$ ½

28. $x^3 - x \geq 0\ \ on\ \ [-1,0]\ \ ; x^3 - x \leq 0\ \ on\ [0,1]\ ; x^3 - x \geq 0\ on\ [1,2\ ]$ ½

$\int_{-1}^{2}|x^3 - x|dx = \int_{-1}^{0}|x^3 - x|dx + \int_0^1|x^3 - x|dx + \int_1^2|x^3 - x|dx$ ½

=$\int_{-1}^{0}(x^3 - x)dx + \int_0^1 -(x^3 - x)dx + \int_1^2(x^3 - x)dx$ 1 ½

$= \frac{11}{4}$ ½

29. $\frac{dy}{dx} + 2ytanx = sinx$

$P = 2tanx \qquad Q = sinx$

$\int Pdx = 2\int tanx\ dx = 2\log|secx| = logsec^2x$ ½

$e^{\int pdx} = e^{logsec^2x} = sec^2x$ ½

General solution is $y\ sec^2x = \int sinx\ \ sec^2x\ dx + C$

$$= \int secxtanx\, dx + C$$

$$y\, sec^2x = secx + C$$ 1

$$y = cosx + Ccos^2x$$

Putting $y = 0$ and $x = \frac{\pi}{3}$ we get $C = -2$ ½

Required solution is $y = cosx - 2cos^2x$ ½

OR

$(x + y)(dx - dy) = dx + dy$

$(x + y - 1)dx = (x + y + 1)dy$ ½

$\frac{dy}{dx} = \frac{(x+y-1)}{x+y+1}$ ½

Put $x + y = t$; $\frac{dy}{dx} = \frac{dt}{dx} - 1$ ½

$\frac{dt}{dx} - 1 = \frac{t-1}{t+1}$ ½

Separating variables and integrating

$\int \frac{t+1}{t}\, dt = 2\int dx$ ½

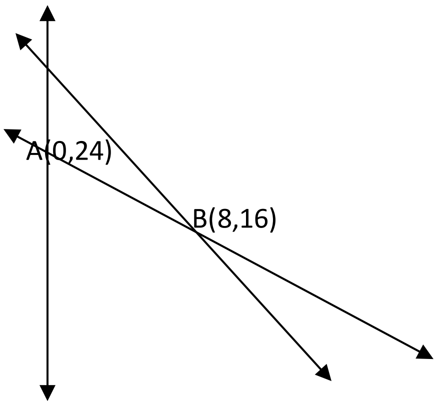
$x + y + \log|x + y| = 2x + C$

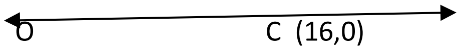
$y - x + \log|x + y| = C$ is the required solution. ½

30. Feasible region with corner points O $(0,0), A(0,24), B(8,16), C(16,0)$

Corner points	Z=300x+190y
(0,0)	0
(0,24)	4560
(8,16)	5440
(16,0)	4800

Z is maximum at (8,16) and maximum value is 5440





31. Let X denote the number of milk chocolate drawn

$P(X=0)=\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}$ ½

$P(X=1)=\left(\frac{2}{6}\times\frac{4}{5}\right)\times 2=\frac{16}{30}$ ½

$P(X=2)=\frac{2}{6}\times\frac{1}{5}=\frac{2}{30}$ ½

X	0	1	2
$P(X)$	$\frac{12}{30}$	$\frac{16}{30}$	$\frac{2}{30}$

½

OR

$P(A)=\frac{2}{7}$; $P(A')=1-\frac{2}{7}=\frac{5}{7}$ ½

$P(B)=\frac{4}{7}$; $P(B')=1-\frac{4}{7}=\frac{3}{7}$ ½

Probability of one of them coming to school on time $=P(A)P(B')+P(A')P(B)$

$=\frac{2}{7}\times\frac{3}{7}+\frac{5}{7}\times\frac{4}{7}=\frac{26}{49}$ 1

SECTION D

32. $x-y+2z=1$; $2y-3z=1$; $3x-2y+4z=2$

The above system cn be written in matrix form $AX=B$ where

$A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ $B=\begin{bmatrix}1\\1\\2\end{bmatrix}$ $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ ½

Let C= $\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$

$AC=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$ 1 ½

$$A^{-1} = C$$

$X = A^{-1}B = CB$ ½

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$ 2 ½

$$x = 0 \ ; y = 5 \,; z = 3$$

33 Reflexive:-Let (a,a) $\in$ R $\Rightarrow |a-a|$ =0, which is a multiple of 4

$\therefore$R is reflexive 1mark

Symmetric: Let (a,b) $\in$ R $\Rightarrow |a-b|$ is a multiple of 4

$|b-a| = |-(a-b)| = |a-b|$ is a multiple of 4 1 mark

Transitive: Let (a,b),(b,c) $\in$ R

$\Rightarrow |a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4

$\Rightarrow$(a-b) = 4p and (b-c) = 4k

$|a-c| = |a-b+b-c| = |4(p+k)|$ which is multiple of 4

$\therefore$ R is transitive 1 mark

Hence R is an equivalence relation

34

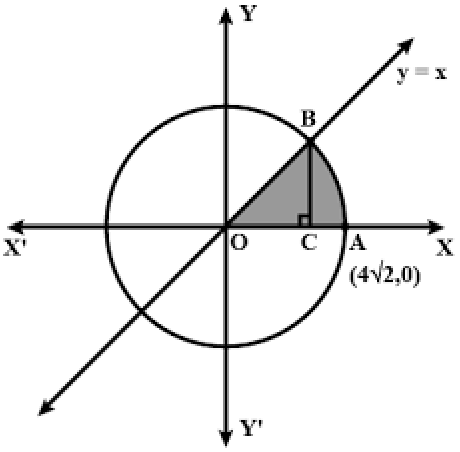


Figure 1 Mark

solving between y=x and $x^2+y^2 = 32$ we get $x = 4\sqrt{2}$ 1 Mark

Required area = $\int_0^{4\sqrt{2}} line - circle$ 1/2 Mark

= $\int_0^{4\sqrt{2}}(y - \sqrt{32 - x^2})$ dx ½ Mark

=8 +(4 π -8) 2 Marks

= 4 π sq units

35 Given $\overrightarrow{a_1}$ = $\vec{i}$ +2 $\vec{j}$+$\vec{k}$ $\overrightarrow{b_1}$ = $\vec{i}$ - $\vec{j}$+$\vec{k}$

$\overrightarrow{a_2}$ =2 $\vec{i}$ - $\vec{j}$ - $\vec{k}$ $\overrightarrow{b_2}$ =2 $\vec{i}$ + $\vec{j}$+2$\vec{k}$ (1Mark)

$\overrightarrow{a_2}$ - $\overrightarrow{a_1}$ = $\vec{i}$ -3 $\vec{j}$ - 2$\vec{k}$ (1/2 Mark)

Formula for Shortest distance (1/2 Mark)

$$\overrightarrow{b_1} \text{ X } \overrightarrow{b_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\,\vec{i} + 3\,\vec{k}$$ (1 Mark)

$\left|\overrightarrow{b_1} \text{ X } \overrightarrow{b_2}\right| = 3\sqrt{2}$ (1 Mark)

Shortest distance = $\frac{3\sqrt{2}}{2}$ (1 mark)

SECTION E

36 (i) $\frac{2x^3}{3} + \frac{4}{3}\pi\, r^3$ 1 mark

(ii)x = $\sqrt{\frac{k^2-4\pi r^2}{6}}$ 1 mark

(iii) r = $\sqrt{\frac{k^2}{54+4\pi}}$ 2 mark

37 (i) Arun's position = (x, x^2+7) (1 Mark)

(ii) Distance = $\sqrt{(x-3)^2 + x^4}$ (1 Mark)

(iii) D = $\sqrt{(x-3)^2+x^4}$ (2 Marks)

$D'(x) = 2x^3 + x - 3 = 0 \rightarrow x = 1$

Clearly D”(x) > 0 at x = 1

$x = 1$ then $= 8$, hence required point (1,8)

38. i)After 5 secons the position of the rocket will be

$x = 3t = 15$;

$y = -4t = -20$

$z = t = 5$ ½

Point is $(15, -20, 5)$ ½

Distance from origin is $\sqrt{(15-0)^2 + (-20-0)^2 + (5-0)^2}$ =$\sqrt{650}$km 1

ii)Given position of the rocket at a time is $(5, -8, 10)$

height of the rocket from the ground = distance between the points $(5, -8, 10)$ and $(5, -8, 0)$ ½

$= \sqrt{(5-5)^2 + (-8+8)^2 + (10-0)^2}$ 1

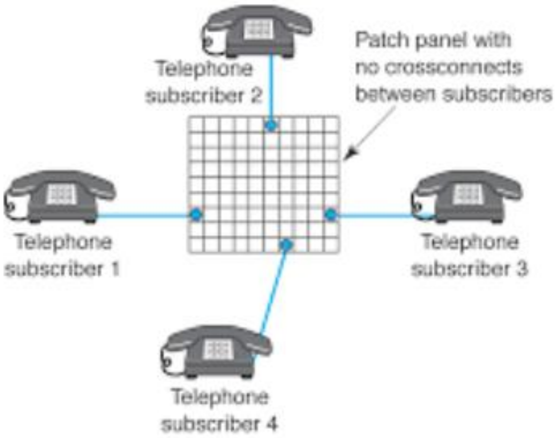
$= 10$ km ½

	KENDRIYA VIDYALAYA ERNAKULAM REGION SAMPLE PAPER 2 (2022-23) MATHEMATICS MARK 80 CLASS XII TIME 3 HOURS	
	GENERAL INSTRUCTIONS: 1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions. 2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each. 3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each. 4. Section C has 6 Short Answer (SA)-type questions of 3 marks each. 5. Section D has 4 Long Answer (LA)-type questions of 5 marks each. 6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.	
	SECTION A (Multiple Choice Questions) **Each question carries 1 mark**	
1	If A=$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ then A^5 is (a) 5A (b) 10A (c) 16A (d) 32A	1
2	If A^2-A+I =0 then the inverse of A is (a)A+I (b)I-A (c) A-I (d)I+A	1
3	If A is a square matrix of order 3 such that $\lvert A \rvert$ = 3 then the value of $\lvert \text{adj(adjA)} \rvert$ (a)9 (b) 81 (c) 6 (d) 27	1
4	If area of triangle is 35 sq. units with vertices (2,-6), (5,4) and (k,4) then k is (a) 12 (b) -2 (c) -12,-2 (d) 12,-2	1
5	$\begin{vmatrix} 5^1 & 5^2 & 5^3 \\ 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \end{vmatrix}$ is equal to (a)0 (b) 5^0 (c) 5^9 (d) 5^{12}	1
6	Find the value of $\frac{d}{dx}(\frac{cos^2x-sin^2x}{cosx+sinx})$ (a) Sinx +cosx (b) cosx – sinx (c) –sinx –cosx (d) sinx -cosx	1
7	If f(x) $= \begin{cases} mx+1, \text{ if } x\leq \frac{\pi}{2} \\ \text{Sinx} +n, x> \frac{\pi}{2} \end{cases}$ is continuous at x =$\frac{\pi}{2}$ then (a)m =1,n=0 (b) m =$n\frac{\pi}{2}$ +1 (c) n= $m\frac{\pi}{2}$+1 (d) n =m =$\frac{\pi}{2}$	1

8	The value of $\int_{1(}^{2} sin^{-1}x + cos^{-1}x)$ dx is (a)$\frac{\pi}{2}$ (b) -$\frac{\pi}{2}$ (c) 0 (d) 2π	1
9	Evaluate$\int \frac{1}{sin^2xcos^2x}$ dx (a) tanx + cotx (b) tanx- cotx (c) 1 (d) -1	1
10	Determine the sum of order and degree of the differential equation $\sqrt{1+(\frac{dy}{dx})^2}$ = 3x - $\frac{dy}{dx}$ (a) 2 (b) 1 (c) 3 (4) 4	1
11	Solve the differential equation cos($\frac{dy}{dx}$) = a, a$\in R$. (a) y =$\frac{-1}{\sqrt{1-a^2}}$ +C (b) y = x cos^{-1}a +C (c) x= y cos^{-1}a+ C (d) y =$\frac{1}{\sqrt{1-a^2}}$ +c	1
12	$\vec{a}$ and $\vec{b}$ unit vectors and θ is the angle between them then $\vec{a} + \vec{b}$ is a unit vector if θ is equal to (a) $\frac{-2\pi}{3}$ (b) $\frac{-\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$	1
13	$\vert\vec{a}$ x $\vec{b}$ $\vert^{2}$ + $\vert\overrightarrow{a}.\vec{b}$ $\vert^{2}$ =400 , $\vert\vec{a}\vert$ = 5 then find the value of $\vert\vec{b}\vert$ (a) 5 (b) 4 (c) $5\sqrt{2}$ (d) $\sqrt{2}$	1
14	Find the area of a parallelogram whose one diagonal is 2i+j-2k and one side is 3i +j –k (a) $\sqrt{21}$ (b) $\sqrt{6}$ (c) $\frac{\sqrt{21}}{2}$ (d) $\frac{\sqrt{6}}{2}$	1
15	If objective function Z =px +qy is maximum at (4,-2) and maximum value is 10 such that p = 3q then find p &q (a) P = 3, q=1 (b) p = -3 q = -1 (c) p=3 , q= -1 (d) p =-3 , q= 1	1
16	A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively then cos 2α + cos 2β + cos 2γ is equal to (a) -1 (b) 1 (c) 3 (d) 2	1
17	The point which does not lie in the half-plane 2x + 3y -12 < 0 is: (a). (2,1) (a). (1,2) (c). (-2,3) (d). (2,3)	1
18	An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black? (a). 3/7 (b). 7/3 (c). 1/7 (d). ⅓	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A.	

	(b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	Assertion (A) Range of tan^{-1} x is $(-\frac{\pi}{2}, \frac{\pi}{2})$ Reason (R) Domain of tan^{-1} x is R	1
20	Assertion (A) The position vector of a particle in a rectangular coordinate system is (3,2,5) then its position vector is $3\hat{\imath}+5\hat{\jmath}+3\hat{k}$ Reason (R) the displacement vector of the particle that moves from (2,3,5) to the point (3,4,5) is $\hat{\imath}+\hat{\jmath}$	1
	SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each	
21	Find the value of sin^{-1} (cos $\frac{3\pi}{5}$) OR Prove that the f $R \to R$ $defined\ by$ f (x) = x^3 + 4 is one one and onto	2
22	Water is leaking from a conical funnel at the rate of 5 cubic centimeter per second . if the radius of the base of the funnel is 10 cm and the altitude is 20 cm ,find the rate at which water level is dropping when it is 5cm from the top	2
23	If $\vec{a} = \hat{\imath} + \hat{\jmath} - 5\hat{k}$ and $\overrightarrow{b}$ $=\hat{\imath}-4\hat{\jmath}+3\hat{k}$ find a unit vector parallel to $\vec{a}+\overrightarrow{b}$ Or Find the directions cosines of a line passing through the origin and lying in the first quadrant ,making equal angle with the three coordinate axis	2
24	Solve x log x $\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x	2
25	If $\|\vec{a}\| = 4$,$\|\vec{b}\| = 3$ and $\overrightarrow{a}.\vec{b} = 6\sqrt{3}$ find $\|\vec{a}$ x $\vec{b}\|$	2
	SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)	
26	Evaluate $\int \frac{(x+2)\,dx}{\sqrt{(x-2)(x-3)}}$	3
27	$Evaluate\ \int tan^{-1}$ x dx	3
28	$Evaluate\ \int_0^{\frac{\pi}{2}}(2logsinx - logsin2x)dx$ OR $\int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-sinx}{1-cosx}\right)$ dx	3
29	Solve $(y+x^2y)\frac{dy}{dx} = 3x + xy^2$ OR $X^2\frac{dy}{dx} = x^2 + 5xy + 4y^2$	3
30	Maximise Z = 80x +120y subject to constraints 9x +12y ≤ 180 , $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0\ y \geq 0.$	3

31	Bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown if 1 or 2 appears, bag A is chosen otherwise bag B. If two balls are drawn at random without replacement from the selected bag find the probability of getting one red and one black. OR In a game, a man wins Rs 5 for 6 and loses rupees one for any other number, when a fair die is thrown. The man decided to throw a die thrice but quits as and when he gets a six. Find the expected value of the amount he wins/loses.	3
	SECTION D (This section comprises of long answer-type questions (LA) of 5 marks each)	
32	IF A = $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ B = $\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ are two square matrices find AB Hence solve x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7	5
33	Find the area of the region { (x,y) : $x^2 + y^2 \le 4$, x + y ≥ 2 } using integration	5
34	Let N be the set of all natural numbers and R be a relation defined by $(a, b)\ R\ (c, d)_{)}$ If bc(a+d) = ad(b+c). show that R is an equivalence relation	5
35	Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point (1,3,3) OR Find the foot of the perpendicular from (0,2,3) on the line $\vec{r} = -3\hat{\imath} + \hat{\jmath} - 4\hat{k} + k(5\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$	5
	SECTION E This section comprises of 3 case-study/passage-based questions of 4 mark each with subpart.	
36	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = 4x - $\frac{1}{2}x^2$ where x is the number of days exposed to sunlight. (i) The rate of growth of the plant with respect to sunlight is (ii) What is the number of days it will take for the plant to grow to the maximum height? (iii) What is the maximum height of the plant? (iv) If the height of the plant is 7/2 cm, the number of days it has been exposed to the sunlight is	4
37	A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of 1 one subscriber will discontinue the service.	4



(i) If x be the annual subscription then the total revenue of the company after increment will be

(ii) How much fee the company should increase to have maximum profit

38 One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time. If leap year is considered, then answer the following questions. 4

(i) The probability that it rains on chosen day is
(ii) The probability that it does not rain on chosen day is
(iii) The probability that the weatherman predicts correctly is
(iv) The probability that it will rain on the chosen day, if weatherman predict rain for that day, is

	ANSWERS	
1	16A	
2	I-A	
3	81	
4	12,-2	
5	0	

6	-cosx-sinx	
7	n =$\frac{m\pi}{2}$	
8	$\frac{\pi}{2}$	
9	tanx-cotx +c	
10	3	
11	Y = x $\cos^{-1}$a+c	
12	$\frac{2\pi}{3}$	
13	4	
14	$3\sqrt{2}$	
15	P =3, q= 1	
16	-1	
17	(2,3)	
18	$\frac{3}{7}$	
19	b	
20	d	
21	$\frac{-\pi}{10}$ OR Proof	
22	$\frac{4}{45\pi}$	
23	$\frac{2\hat{\imath} - 3\hat{\jmath} - 2\hat{k}}{\sqrt{17}}$ or $\pm(\frac{1}{\sqrt{3}}), \pm(\frac{1}{\sqrt{3}}), \pm(\frac{1}{\sqrt{3}})$	
24	Y log x = $\frac{-2}{x}$ (1+logx) +c	
25	6	
26	I =$\sqrt{x^2 - 5x + 6}$ + $\frac{9}{2}$ log(x- $\frac{5}{2}$ +$\sqrt{x^2 - 5x + 6}$) +C	
27	x $\tan^{-1}$x – ½ log(1+x^2) +C	
28	$-\frac{\pi}{2}$log2 OR $e^{\frac{\pi}{2}}$	
29	3+y^2 = c(1+x^2) OR $\frac{-x}{2(x+2y)}$ = logx +c	
30	Max Z= 1680 and max point is (12,6)	
31	P(E1) = $\frac{2}{6}$, P(E2) = $\frac{4}{6}$, P(A/E1) = $\frac{24}{45}$ P(A/E2) = $\frac{21}{45}$,P(A) = $\frac{22}{45}$	

	OR X = 4,3,-3 P(x=5) =$\frac{1}{6}$, P(x=4) =$\frac{5}{36}$, P(x=3) =$\frac{25}{216}$, P(x=-3) =$\frac{125}{216}$,E(x)=0	
32	X= 2, y= -1, z= 4	
33	Required Area =$\int_0^2 \sqrt{4-x^2}\ dx - \int_0^2 (2-x)dx$ =$\pi - 2$	
34	Proof	
35	Points are (-2,-1,3) and (4,3,7) OR K =1, Point is (2,3,-1)	
36	(i) 4 – x (ii) 4 (iii) 8cm (iv) 1	
37	(i) R(x)= (500-x)(300+x) = $-x^2$+200x+150000 (ii) 100	
38	(i) 1/61 (ii) 60/61 (iii) 4/5 (iv) 0.94	

Sample Question Paper - 3
Class XII
Session 2022 -23
Mathematics(Code-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

1. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
2. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
3. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
4. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
5. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each**)** with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 mark

1. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, then a+b+c is

a) -2 b) 0 c) -3 d) -5

2. If A is a square matrix of order 3, $|A'| = 5$,then $|A^{-1}| =$

a) 5 b) 0 c) -5 d) 1/5

3. Let the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 3 \; and \; |\vec{b}| = \sqrt{2}/3$,then $\vec{a}$ x $\vec{b}$ is a unit vector if the angle between them is

a) 30^0 b) 45^0 c) 60^0 d) 90^0

4. If the function f(x)=$\begin{cases} 3x - 8, if \;\; x \leq 5 \\ 2k \; , if \;\; x > 5 \end{cases}$ is continuous , then the value of k is;

a) 2/7 b) 7/2 c) 3/7 d) 4/7

5. The antiderivative of $\frac{1}{\sin^2 x\ \cos^2 x}$ equals

a) tan x + cot x + c b) tan x- cot x +c c) tan x .cot x+ c d) tan x +c

6. If m and n are the order and degree of the differential equation $(y^{l})^2 + y^{ll} + y=0$, then find m + n

a) 2 b) 3 c) 0 d) 4

7. In an LPP, if the objective function Z= ax +by has the same maximum value on two corner points of the feasible region, then the number of points at which maximum Z occurs is

a) 0 b) 2 c) finite d) infinite

8. The unit vector perpendicular to vectors i-j and i+j forming a right handed system is

a) k b) i -k c) ½(i-j) d) 2 (i+j)

9. The value of $\int_0^1 \frac{dx}{1+x^2}$ is

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{4}$ d)0

10. If A= $\begin{bmatrix} x & 2 \\ 2 & x \end{bmatrix}$ and $|A^3| = 27,$ then find the value of x

a) 1 b)± 2 $c) \pm\sqrt{5}$ $d) \pm\sqrt{7}$

11. If the corner points of the feasible region of an LPP are (0,3), (3,2) and (0,5), then the minimum value of Z= 11x+ 7y is:

a) 21 b) 33 c) 14 d) 35

12. For the determinant $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, $a_{11}A_{21} + a_{12}A_{22} + a_{13} A_{23}$ is:

a) 23 b) 0 c)-3 d) 3

13. Given that A is a square matrix of order 3 and lAl = - 5 then l adj Al is ;

a) 5 b) -5 c) 25 d) -25

14. If P(A)=1/2 , P(B)= 0 then P(A/B) is:

a) 0 b) ½ c) not defined d) 1

15. The number of arbitrary constants in the particular solution of a differential equation of third order is:

a) 3 b) 2 c) 1 d) 0

16. If $y = e^{-x}$, then y^{ll} is :

a) -y b) y c) x d) -x

17. If $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1, then\ \ |\vec{a}-\vec{b}|$ is equal to :

a) 1 b) $\sqrt{2}$ $c) \sqrt{3}$ $d)$ 0

18. The value of μ for which the lines $\frac{x-1}{1} = \frac{y-2}{\mu} = \frac{z+1}{-1}\ and\ \frac{x+1}{-\mu} = \frac{y+2}{2} = \frac{z-2}{1}$ are perpendicular to each other is:

a) 0 b)-1 c) 1 d) 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement ofReason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion(A): Principal value of $\cos^{-1}(1)$ is π
Reason(R) : Value of $\cos 0^0$ is 1

20. Assertion(A):
The angle between the lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90^0
Reason (R): skew lines in different planes which are parallel and intersecting

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Check whether the function f: R →R defined by f(x)= 4+3cosx is one one -onto

OR

Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

22. An edge of a variable cube is increasing at the rate of 3cm/s. How fast is the volume of the cube increasing when the edge is 10cm long

23. Find the vector and cartesian equations of the line that passes through the points (3,-2,-5) and (3,-2,6)

OR

The two adjacent sides of a parallelogram are 2i- 4j +5k and i-2j-3k. Find the unit vector parallel to its diagonal.

24. Find $\frac{dy}{dx}$ if x= a sec t and y= b tan t at $t= 30^0$

25. Show that the vector i+j+k is equally inclined with the coordinate axes.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q26. Find: $\int \frac{dx}{5-8x-x^2}$

Q27. Two cards are drawn at random with replacement from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution

OR

Probabilities of solving a specific problem independently by A and B are ½ and $^1/_3$ respectively. If both try to solve problem independently ,then find the probability that
(i) the problem is solved
(ii) exactly one of them solves the problem

Q28. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1 + tanx)dx$

OR

Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + cosx}{9+16\, sin2x} \quad dx$

Q29. Find the particular solution of the differential equation log$\left[\frac{dy}{dx}\right]$ =3x + 4y, given that y=0 when x=0

OR

Solve the differential equation: x cos$\left(\frac{y}{x}\right)$ $\frac{dy}{dx}$ = y cos$\left(\frac{y}{x}\right)$ + x ; x ≠0

Q30. Solve the following Linear Programming Problem graphically: Maximize Z = x + 2y subject to x + 2y ≥ 100, 2x -y ≤ 0, 2x+y ≤ 200, x,y ≥ 0

Q31. Find $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Q32. Make a rough sketch of the region {(x, y) : y ≥ x^2 and y = | x|} and find the area of the region using integration.

Q33.Show that the relation S in the set A = { x ∈ Z : 0 ≤ x ≤ 12} given by S = { (a, b) : a,b ∈ Z, | a – b | is divisible by 4} is an equivalence relation .Also find the equivalence class [3]

OR

Consider f : R - $\left\{\frac{-4}{3}\right\}$ → R - $\left\{\frac{4}{3}\right\}$ given by f(x)= $\frac{4x+3}{3x+4}$ .Show that f is bijective. Also find x such that f(x)=2

Q34.Find the shortest distance between the lines whose vector equations are $\vec{r}$= (1 - t) $\hat{\imath}$ + (t - 2)$\hat{\jmath}$ + (3 - 2t) $\hat{k}$ and $\vec{r}$= (s + 1) $\hat{\imath}$ + (2s - 1)$\hat{\jmath}$ - (2s + 1) $\hat{k}$

OR

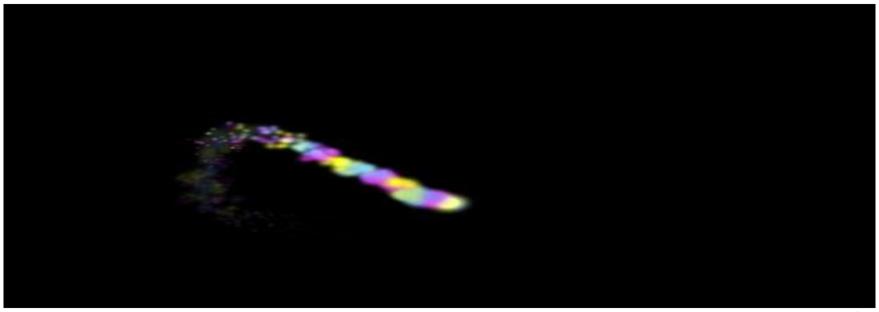
Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3)

Q35. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations x – y + 2z = 1 , 2y – 3z = 1 , 3x – 2y + 4z = 2

SECTION E

(This section comprises of with two sub-parts. First two case study questions have three sub of marks 1, 1, 2 respectively. The third case study question has two sub marks each.)

Q36. Case-Study 1: Read the following passage and answer the questions given below.



A particle is moving on the path given by $f(x) = (x-2)^4(x+1)^3$

(i) Find the critical points of the function f(x)

(ii) Find the point of inflection , if any. Justify your answer

(iii) For what values of x, the function f(x) is decreasing and increasing

OR

(iii) For the function f(x),find the points of local maxima and minima and also find the absolute maxima and minima values of f(x) in [1,3]

Q37. Case-Study 2: Read the following passage and answer the questions given below.

An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier ,placed at (3,7) wants to shoot down the helicopter when it is nearest to him

(i) If (a , b) be the position of position of the helicopter on the curve $y = x^2 + 7$,then find the distance function from soldier to helicopter in terms of 'a'

(ii) Find the critical point of the function

(iii)Use first derivative test to find the position (a, b) that minimise the distance

OR

(iii)Use second derivative test to find the position (a,b) that minimise the distance

Q38. Case-Study 3: Read the following passage and answer the questions given below.

The reliability of a HIV test is specified as follows:

Of people having HIV,90% of the test detect the disease but 10% go undetected. Of people free of HIV,99% of the test are judged HIV -ive but 1% are diagnosed as showing HIV +ive .From a large population of which only 0.1% have HIV, one person is selected at random ,given the HIV test ,and the pathologist reports him/her as HIV +ive.

(i) What is the probability that the person's HIV test is diagnosed as +ive

(ii) What is the probability that the person actually has HIV

ANSWERS

1)-5

2) 1/5

3) 45^0

4) 7/2

5) tan x-cot x +c

6) 3

7) infinite

8) k

9) 45^0
10)$\pm\sqrt{7}$
11) 21
12) 0
13) 25
14) not defined
15) 0
16) y
17) $\sqrt{3}$

18) 1
19) A is false but R is true
20) A is true but R is false
21) Neither one one nor onto **OR** 1
22) 900 cm^3/s
23) $\vec{r}$ = 3i-2j-5k + t(11k) , $\frac{x-3}{0}=\frac{y+2}{0}=\frac{z+5}{11}$ **OR** $\frac{3i-6j+2k}{7}$
24) 2b/a
Q.26 $\frac{1}{2\sqrt{21}}$ log $\left|\frac{\sqrt{21}+4+x}{\sqrt{21}-4-x}\right|$ +C
Q.27

X	P(X)
0	1/4
1	2/4
2	1/4

mean = 1 **OR** (i) 2/3 (ii) ½
Q.28 $\frac{\pi}{8}$ log 2 **OR** $\frac{1}{40}$log 9
Q.29 $4\,e^{3x}+3\,e^{-4y}-7=0$ **OR** sin$\left(\frac{y}{x}\right)$= log |Cx|
Q.30 max Z =400 at (0,200)
Q.31 $\frac{-1}{3}\,tan^{-1}\,x$ + $\frac{2}{3}\,tan^{-1}\,\frac{x}{2}$ + C
Q.32 $^1/_3$
Q.33 [3]={3,7,11} **OR** x = $\frac{-5}{2}$
Q.34 $\frac{8}{\sqrt{29}}$ **OR** (-2,-1,3) or (4,3,7)
Q.35 x = 0 ,y = 5 ,z = 3
Q.36 (i) x= 2,-1 or 2/7
(ii) x $= -1$,as the values of x varies through -1,sign of f'(x) does not change
(iii) f(x) increases on (-$\infty, 2/7)$ and $(2,\infty)$
f(x) decreases on (2/7, 2)

OR

point of local minima= 2
point of local maxima = 2/7
absolute maximum value of f = 64 at x =3
absolute minimum value of f = 0 at x =2

Q.37 (i) D= $\sqrt{(a-3)^2\ +a^4}$
(ii) a= 1
(iii) When helicopter is at (1,8) , the nearest distance is $\sqrt{5}$ unit from the soldier
Q.38 (i)0.01089
(ii)90/1089=0.083